CHINESE JOURNAL OF PHYSICS VOL. 10, No. 1 APRIL, 1972

The Nuclear Mass Formula and the Independent Particle Model

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(Received 3 March 1972)

The attempt of Lee and Green to account for the experimental nuclear mass surface is re-examined in a simplified manner by studying the energy level sequence of the Malenka-Green potential and that of the Wood-Saxon potential both with a spin orbit term.

When the energy sum is fitted to the Bethe-Weizsäcker mass formula, the MG potential gives a larger volume energy term a, and a nearly exact surface energy term a_s; the WS potential also gives a larger a, but a much smaller a_s. However. the symmetry dependence of the depth of the WS potential proposed by Veje can yield a factor $[1-\kappa(N-Z)^2/A^2]$ in the Myers-Swiatecki mass formula.

§ 1. INTRODUCTION

THE general properties of nucleus can be accounted for by the nuclear potential acting upon the nucleons. Many efforts have been made to establish a family of potentials from fundamental theories.[1] However, a complete success has not been achieved at present. Therefore a phenomenological form of potentials should be introduced which includes several adjustable parameters. A ready method of a test for the validity of the family of potentials with such a choice is to derive a nuclear mass formula from the independent particle model (IPM), by studying the nuclear energy levels calculated from the potentials.

§2. NUCLEAR POTENTIALS

Lee and Green[2] attempted to determine the extent to which the IPM can account for the expermental nuclear mass surface. The approximate eigenvalues for a spherical well with an exponential diffuse boundary-called the Malenka-Green (MG) potential-were applied by them. The purpose of this article is to re-examine the same attempt in a simplified manner by studying the energy level sequence of the MG potential and that of the Wood-Saxon (WS) potential both with a spin orbit term.

The eigenvalues for the MG potential

(1) For example. P. J. Siemens. Nucl. Phys. **A141**, 225 (1970).
(2) A. E. S. Green. T. Sawada and D. S. Saxon. ***The Nuclear Independent Particle Model*** (Academic Press, N.Y. and London. 1968).

$$V(r) = -V_0, \qquad r<a$$

$$V(r) = -V_0 \exp\left(-\frac{r-a}{\delta a}\right), \qquad r>a$$

with the Thomas-Frenkel spin orbit term

$$V_{so} = -a_{so}^2 \left|\frac{1}{r}\frac{dV}{dr}\right| \frac{L\cdot S}{\hbar^2}$$

were computed against any nucleon number A by Green[2] for the values

$$V_0 = 40\ \text{MeV}, \qquad a_{so} = 1\ \text{fm},$$

$$\delta = d/a, \qquad d = 1\ \text{fm}' \ \text{and} \qquad a = 1.32\,A^{1/3} - 0.8\ \text{fm}.$$

Similarly, those for the Wood-Saxon potential

$$V(r) = -V_0 f(r) = \frac{-V_0}{1+\exp\left(\frac{r-R}{a}\right)}$$

with the spin orbit term

$$V_{so} = -V_1 \frac{(L\cdot S)}{\hbar^2} r_0^2 \frac{1}{r}\frac{d}{dr} f(r)$$

were computed against A by Veje[3] for the values

$$V_0 = \left(51 - 33\frac{N-Z}{A}\right)\ \text{MeV},$$

$$V_1 = 0.44\,V_0 = \left(22 - 14\frac{N-Z}{A}\right)\ \text{MeV},$$

$$R = r_0 A^{1/3}, \qquad r_0 = 1.27\ \text{fm} \qquad \text{and} \qquad a = 0.67\ \text{fm}.$$

In these computations they assumed according to custom that protons sense the same nuclear potential as that seen by neutrons and that the Coulomb potential represents the only difference. Using these eigenvalues the energy sum $E(A)$ is computed for the nucleus on the line of beta stability. This line is represented by the neutron excess $D=N-Z$ for various A, and is approximated to an empirical formula[2]

$$N-Z = D(A) = \frac{0.40\,A^2}{A+200}.$$

§3. FITS TO THE BETHE-WEIZSÄCKER AND THE MYERS-SWIATECKI MASS FORMULAE

The quantities $-E/A^{2/3}$ are then plotted against $A^{1/3}$, which yield nearly straight lines as shown in Fig. 1. The slope of the straight line and its inter-

(3) A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. 1 (W. J. Benjamin Inc. N. Y. and Amsterdam, 1969) p. 236.

section with the ordinate determine respectively the coefficient of volume energy term a, and that of the surface energy term a, in the Bethe-Weizsäcker formula

$$E(A)=-a_vA+a_sA^{2/3}+\frac{a_cZ^2}{A^{1/3}}+\frac{a_eD^2}{4A}.$$

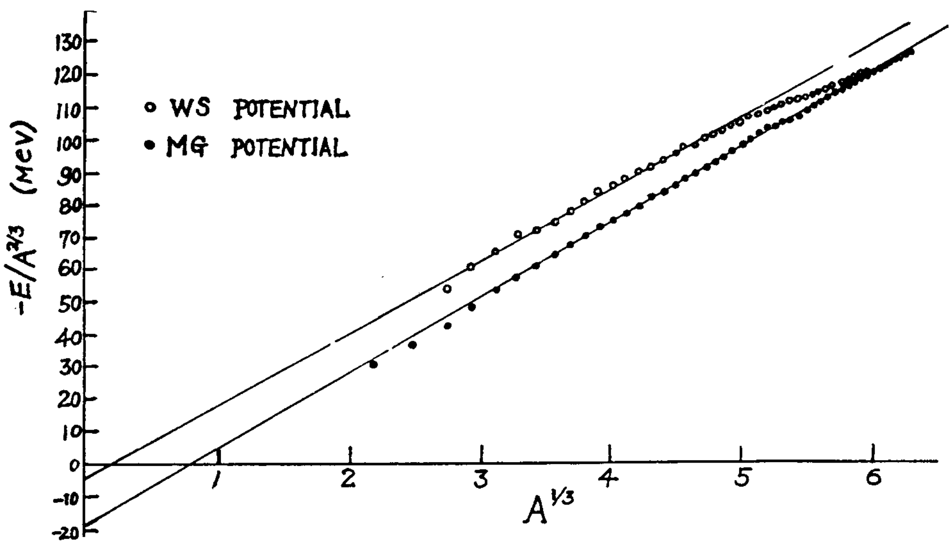

Fig. 1. Plots of $-E/A^{2/3}$ against $A^{1/3}$.

The values of a, and a, determined are compared in Table 1 with the exact one given by Green. The volume energy term given by each of the potentials is larger than the exact one. The MG potential yields a nearly exact surface energy 18.3 MeV, while WS potential gives a much smaller value 4.1 MeV. A slight bend can be seen in the heavy nuclei part for the WS potential. Therefore, the effect of symmetry energy term $a_eD^2/4A$ is examined by plotting values of $(-E/A^{2/3})+a_e[(N-Z)/2A]^2A^{1/3}$ against $A^{1/3}$, where a, is taken to be 94.76 MeV. However, as shown in Fig. 2, no significant improvement is found for the case of WS potential.

Table 1. Calculated volume energy term a, and surface energy term a, and the exact values

	Exact values by Green	MG potential	WS potential
a_v (MeV)	15.82	23.2	22.1
a_s (MeV)	17.90	18.3	4.1

Recently, Myers and Swiatecki[4] have harmoniously blended the liquid drop model and the shell model, and have derived a new type of mass formula

(4) W. D. Myers and W. J. Swiatecki, Ann. Phys. 55, 395 (1969). T. Kodama, Progr. Theoret. Phys. (Kyoto) 45, 1112 (1971); T. Kodama and M. Ynmada. *ibid*, **45, 1763** (1971).

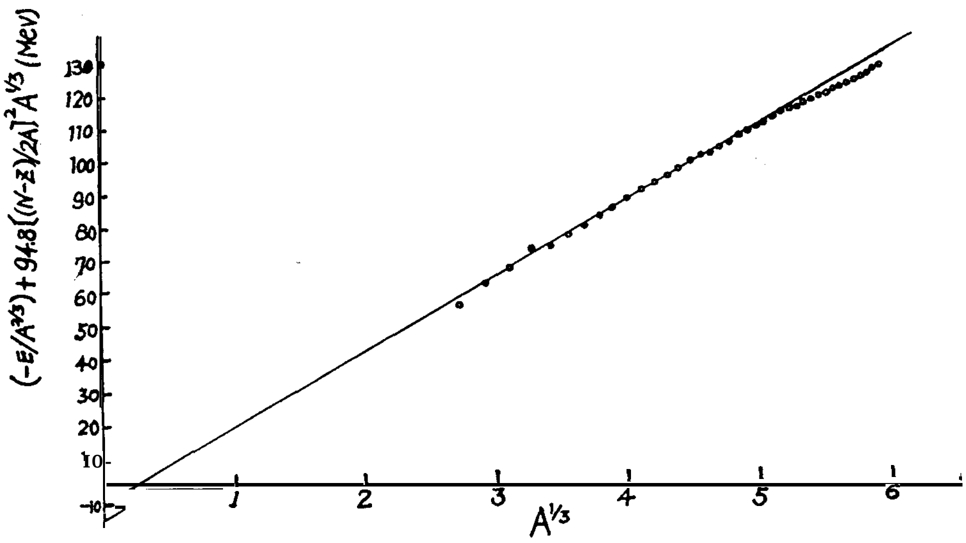

Fig. 2. Values of ($-E/A^{2/3})+a_eD^2/4A$ are plotted against $A^{1/3}$ *to* test the effect of the symmetry energy term $a_eD^2/4A$.

$$E(A) = (-a_vA+a_sA^{2/3})\left[1-\kappa\left(\frac{N-Z}{A}\right)^2\right]+\frac{a_cZ^2}{A^{1/3}}-7\exp\left(-6\frac{|N-Z|}{A}\right),$$

where a_v=15.68 MeV, a_s=18.56 MeV, a_c=0.717 MeV (r_0=1.205 fm) and κ=1.79. Since the last term which represents the shell effects is not significant in the present calculation, it is disregarded together with the Coulomb energy term. The value

$$\left(\frac{-E}{A^{2/3}}\right)\Big/\left[1-1.79\left(\frac{N-Z}{A}\right)^2\right]$$

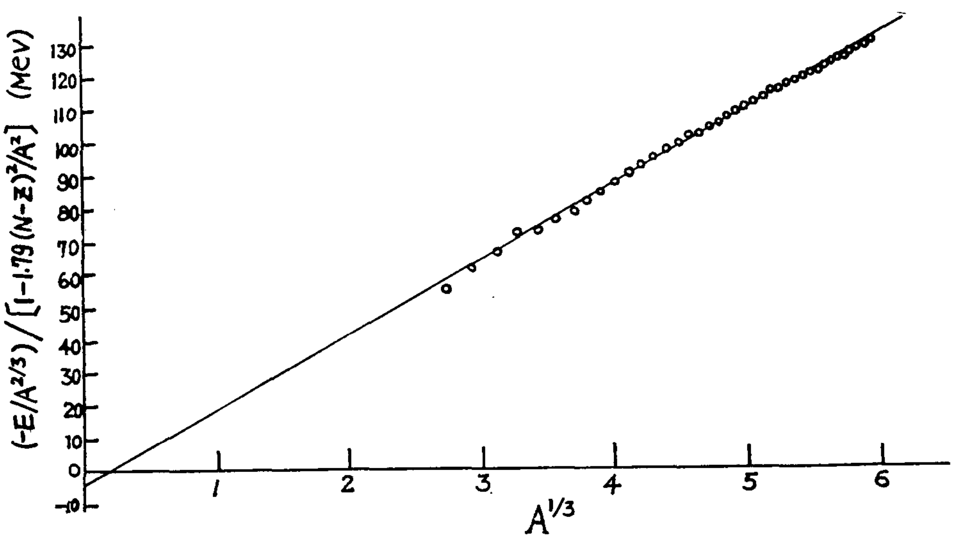

Fig. 3. Plots of $(-E/A^{2/3})/[1-\kappa(N-Z)^2/A^2]$ against $A^{1/3}$. The symmetry dependence of the potential depth introduced by Veje yields the factor $[1-\kappa(N-Z)^2/A^2]$ in the Myers-Swiatecki formula.

is then plotted against $A^{1/3}$ as illustrated in Fig. 3. The slope and the intersection with the ordinate determine respectively the values

$$a_v = 22.8 \text{ MeV} \qquad \text{and} \qquad a_s = 4 \text{ MeV},$$

both of which show no remarkable change from the fit to the Bethe-Weizsäcker formula. However the bend is straighten, and it can be concluded that the symmetry dependence of the potential depth introduced by Veje yields the factor $[1-\kappa\,(N-Z)^2/A^2]$ in the Myers-Swiatecki formula.

§4. CONCLUDING REMARKS

Green[(2)] has demonstrated in his plots of $\varepsilon_0^2-\varepsilon^2$ vs. $\varepsilon_0^2=2ma^2\ V_0/\hbar^2$ for different δ's that the eigenvalues $\varepsilon^2=\hbar^2E/2ma^2$ increase always as ε_0^2 increases. Therefore the larger volume energy term a_v obtained from both types of potential could be remedied by changing the potential extension a or depth V_0. On the other hand, the small surface energy term a_s given by WS potential is probably due to the larger value of spin-orbit term as well as to the inadequacy of the surface thickness parameter.