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Variation of Ultrasonic Velocity and Absorption with Frequency in High Viscosity Oils*

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Being based on the Maxwellian model and neglecting the thermal conduction in a visco-elastic body, the ultrasonic absorption and velocity in liquid are calculated.

The ultrasonic absorption and velocity in high viscosity oils are also measured. The experimental and the theoretical results are compared.

§ 1. INTRODUCTION

A great many observations have been performed on the ultrasonic velocity and absorption of high viscosity liquids and solutions, including polymeric substances[1-5] and vegetable oils[6-11]. The theoretical calculation for absorption and velocity versus $\tau\omega$ (ω: angular frequency, τ: relaxation time) was made by Takizawa[12,13] and Kuo[14].

In the present paper, the author reports the theoretical calculation (being based on the Maxwellian model) for ultrasonic absorption and velocity versus frequency and compared with the experimental results of the frequency dependence of ultrasonic velocity and absorption in rapeseed oil, soybean oil, black sesame oil and cotton seed oil.

§2. THEORY

(1) *Notations*

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x_i: rectangular coordinates, $(i=1, 2, 3)$

ξ_i: components of displacement, $(i=1, 2, 3)$

$\varepsilon_{ij}=\frac{1}{2}\cdot\left(\frac{\partial\xi_j}{\partial x_i}+\frac{\partial\xi_i}{\partial x_j}\right)$: components of strain tensor, $(i, j=1, 2, 3)$

A^0_{ij}: components of stress tensor, no temperature effect being taken into account, $(i, j=1, 2, 3)$

$A_{ij}=A^0_{ij}+A'_{ij}$: components of stress tensor, including explicitly thermal stress, $(i, j=1, 2, 3)$

t: time,

$\rho=\rho_0+\rho'$: density, ρ_0: density at static state,

$p=p_0+p'$: pressure, p_0: static pressure,

$T=T_0+\vartheta$: temperature, T_0: initial temperature,

κ: thermal conductivity,

k_0: static volume modulus,

U: internal energy per unit mass in the macroscopic sense,

λ_r: partial volume moduli, $(r=1, 2, 3, \ldots, l)$

$\lambda'_r=\lambda_r\tau_r$: partial volume viscosities, $(r=1, 2, 3, \ldots, l)$

μ_r: partial shearing rigidities, $(r=1, 2, 3, \ldots, m)$

$\mu'_r=\mu_r\tau^{(1)}_r$: partial shearing viscosities, $(r=1, 2, 3, \ldots, m)$

a_r/k_0: coefficients of partial thermal expansion, $(r=1, 2, 3, \ldots, n)$

τ_r, $\tau^{(1)}_r$, $\tau^{(2)}_r$, $\tau^{(3)}_r$, and $\tau^{(4)}_r$: relaxation times,

C_0: static specific heat at constant volume,

C_r: partial specific heat at constant volume, $(r=1, 2, 3, \ldots, p)$

$\omega=2\pi f$: circular frequency, f: frequency,

v: phase velocity of wave,

a: amplitude absorption of ultrasonic per unit length,

$b_0=-p_0$,

$b^{(r)}_{ij}=2\mu_r\varepsilon_{ij}$,

$$D=\frac{d}{dt}=\frac{\partial}{\partial t}+U_j\frac{\partial}{\partial x_j}=\frac{\partial}{\partial t},$$

$$E=\frac{\lambda_1}{k_0}+\frac{4}{3}\cdot\frac{\mu_1}{k_0}+\frac{a_1 p_0}{k_0 C_0\rho_0}.$$

(2) *Application to supersonic wave*

The fundamental equations of the supersonic waves under no body force, which state the conservation of momentum and of energy, are expressed respectively, in linear forms[12-14]:

$$\rho_0\frac{\partial^2\xi_i}{\partial t^2}=\frac{\partial A_{ij}}{\partial x_j}, \tag{1}$$

and

$$\rho_0 \frac{\partial U}{\partial t} = A_{ij} \frac{\partial \varepsilon_{ij}}{\partial t} + \kappa \Delta \vartheta, \tag{2}$$

where the expression $q_j = -\kappa \frac{\partial T}{\partial x_j}$ $(j=1, 2, 3)$, was used for the heat flux vector.

The eqs. for the stress and the internal energy are expressed respectively[12,14] by

$$A_{ij}(t) = -p_0 \delta_{ij} + \left(k_0 + \sum_{r=1}^{l} \frac{\lambda_r' D}{1+\tau_r D} - \frac{2}{3} \sum_{r=1}^{n} \frac{\mu_r' D}{1+\tau_r^{(1)} D} \right) \cdot \varepsilon_{kk} \delta_{ij} +$$
$$+ 2 \sum_{r=1}^{m} \frac{\mu_r' D}{1+\tau_r^{(1)} D} \cdot \varepsilon_{ij} - \sum_{r=1}^{n} \frac{\alpha_r' D}{1+\tau_r^{(2)} D} \cdot \vartheta \delta_{ij}, \tag{3}$$

and

$$\delta U = \left\{ C_0 + \sum_{r=1}^{s} \frac{C_r \tau_r^{(3)} D}{1+\tau_r^{(3)} D} \right\} \cdot \delta T + \left\{ b_0 \delta_{ij} + \sum_{r=1}^{q} \frac{b_{ij}^{(r)} \tau_r^{(4)} D}{1+\tau_r^{(4)} D} \right\} \cdot \delta \varepsilon_{ij}. \tag{4}$$

Putting the eqs. (3) and (4) for the stress and the internal energy into eqs. (1) and (2), we obtain[12,14], retaining the small quantities of first order:

$$\rho_0 \frac{\partial^2 \xi_i}{\partial t^2} = \left(k_0 + \sum_{r=1}^{l} \frac{\lambda_r' D}{1+\tau_r D} - \frac{2}{3} \sum_{r=1}^{m} \frac{\mu_r' D}{1+\tau_r^{(1)} D} \right) \cdot \frac{\partial \varepsilon_{kk}}{\partial x_i} +$$
$$+ 2 \sum_{r=1}^{m} \frac{\mu_r' D}{1+\tau_r^{(1)} D} \cdot \frac{\partial \varepsilon_{ij}}{\partial x_j} - \sum_{r=1}^{n} \frac{\alpha_r' D}{1+\tau_r^{(2)} D} \cdot \frac{\partial \vartheta}{\partial x_i}, \tag{5}$$

and

$$\rho_0 \left\{ \left(C_0 + \sum_{r=1}^{s} \frac{C_r \tau_r^{(3)} D}{1+\tau_r^{(3)} D} \right) \cdot \frac{\partial \vartheta}{\partial t} + b_0 \delta_{ij} + \sum_{r=1}^{q} \frac{b_{ij}^{(r)} \tau_r^{(4)} D}{1+\tau_r^{(4)} D} \right) \cdot \frac{\partial \varepsilon_{ij}}{\partial t} \right) =$$
$$= -p_0 \frac{\partial \varepsilon_{kk}}{\partial t} + \kappa \Delta \vartheta. \tag{6}$$

If the wave of volume dilatation is considered, the following equation of motion can be obtained:

$$\rho_0 \frac{\partial^2}{\partial t^2} \varepsilon_{kk} = \left(k_0 + \sum_{r=1}^{l} \frac{\lambda_r' D}{1+\tau_r D} + \frac{4}{3} \sum_{r=1}^{m} \frac{\mu_r' D}{1+\tau_r^{(1)} D} \right) \cdot \Delta \varepsilon_{kk} - \sum_{r=1}^{n} \frac{\alpha_r' D}{1+\tau_r^{(2)} D} \cdot \Delta \vartheta. \tag{7}$$

As a special case, we shall take the plane wave, progressing into $+x$-direction with circular frequency ω:

$$\left.\begin{array}{l} x_1 = x, \ \frac{\partial}{\partial x_2} = \frac{\partial}{\partial x_3} = 0, \\ \xi_1 = R_e \xi \cdot \exp\,[i\omega t - \beta x], \xi_2 = \xi_3 = 0, \\ \delta T = \vartheta = R_e \vartheta \cdot \exp\,[i\omega t - \beta x], \\ \varepsilon_{kk} = \varepsilon_{11} = -\beta \xi \cdot \exp\,[i\omega t - \beta x], \mathrm{Im}(\beta) > 0, \\ \varepsilon_{ij} = 0. \qquad \text{(except } i=j=1) \end{array}\right\} \tag{8}$$

The equation of motion (5), and of conservation of energy (6), are:

$$\rho_0 \frac{\partial^2}{\partial t^2}\varepsilon_{11} = \left(k_0 + \sum_{r=1}^{l} \frac{\lambda'_r D}{1+\tau_r D} + \frac{4}{3}\sum_{r=1}^{m} \frac{\mu'_r D}{1+\tau_r^{(1)} D}\right)\cdot \Delta\varepsilon_{11} - \sum_{r=1}^{n} \frac{a'_r D}{1+\tau_r^{(2)} D}\cdot \Delta\vartheta, \tag{9}$$

and

$$\left(C_0 + \sum_{r=1}^{s} \frac{C_{ij}^{(r)}\tau_r^{(3)} D}{1+\tau_r^{(3)} D}\right)\cdot \frac{\partial \vartheta}{\partial t} + \left(b_0 + \sum_{r=1}^{q} \frac{b_{11}^{(r)}\tau_r^{(4)} D}{1+\tau_r^{(4)} D}\right)\cdot \frac{\partial \varepsilon_{11}}{\partial t}$$
$$= -\frac{p_0}{\rho_0}\cdot \frac{\partial \varepsilon_{11}}{\partial t} + \frac{\kappa}{\rho_0}\cdot \Delta\vartheta. \tag{10}$$

Substitutting eq. (8) into eqs. (9) and (10), and neglecting the terms of higher order, eqs. (5) and (6) reduce to:

$$\left\{\rho_0\omega^2 + \left(k_0 + \sum_{r=1}^{l} \frac{i\omega\lambda'_r}{1+i\omega\tau_r} + \frac{4}{3}\sum_{r=1}^{m} \frac{i\omega\mu'_r}{1+i\omega\tau_r^{(1)}}\right)\cdot\beta^2\right\}\cdot\xi + \sum_{r=1}^{n} \frac{i\omega a'_r}{1+i\omega\tau_r^{(2)}}\cdot\beta\vartheta = 0, \tag{11}$$

and

$$-i\omega\beta\left(\frac{p_0}{\rho_0} + b_0 + \sum_{r=1}^{q} \frac{i\omega b_{11}^{(r)}\tau_r^{(4)}}{1+i\omega\tau_r^{(4)}}\right)\cdot\xi + \left\{i\omega\left(C_0 + \sum_{r=1}^{s} \frac{i\omega C_r\tau_r^{(3)}}{1+i\omega\tau_r^{(3)}}\right) - \frac{\kappa}{\rho_0}\cdot\beta^2\right\}\cdot\vartheta = 0. \tag{12}$$

Considering that the simultaneous equations (11) and (12) have nontrivial solution, it can be seen that β is to satiafy the following equation:

$$\begin{vmatrix} \rho_0\omega^2 + A\beta^2, & \displaystyle\sum_{r=1}^{n} \frac{i\omega a'_r}{1+i\omega\tau_r^{(2)}}\cdot\beta \\ -i\omega\beta B, & i\omega\left(C_0 + \displaystyle\sum_{r=1}^{s} \frac{i\omega C_r\tau_r^{(3)}}{1+i\omega\tau_r^{(3)}}\right) - \dfrac{\kappa}{\rho_0}\cdot\beta^2 \end{vmatrix} = 0, \tag{13}$$

i.e.

$$-\frac{\kappa}{\rho_0}\cdot A\beta^4 + \left\{\left(i\omega C_0 - \sum_{r=1}^{s} \frac{\omega^2 C_r\tau_r^{(3)}}{1+i\omega\tau_r^{(3)}}\right)A - \kappa\omega^2 - \right.$$
$$\left. - \sum_{r=1}^{n} \frac{\omega^2 a'_r}{1+i\omega\tau_r^{(2)}}\cdot\right\}B\cdot\beta^2 + i\omega\left(\rho_0 C_0 + \sum_{r=1}^{s} \frac{i\omega C_r\tau_r^{(3)}}{1+i\omega\tau_r^{(3)}}\right) = 0, \tag{14}$$

with

$$A = k_0 + \sum_{r=1}^{l} \frac{i\omega\lambda'_r}{1+i\omega\tau_r} + \frac{4}{3}\sum_{r=1}^{m} \frac{i\omega\mu'_r}{1+i\omega\tau_r^{(1)}}, \tag{15}$$

and

$$B = \frac{p_0}{\rho_0} + b_0 + \sum_{r=1}^{q} \frac{i\omega b_{11}^{(r)}\tau_r^{(4)}}{1+i\omega\tau_r^{(4)}}. \tag{16}$$

In this equation, if we put $\kappa = 0$, which approximately hold for high viscosity oils, then we obtain:

$$\beta^2=-\frac{\omega^2}{v_0^2}\div\Bigg[1+\sum_{r=1}^{l}\frac{i\omega\lambda_r'}{k_0(1+i\omega\tau_r)}+\frac{4}{3}\sum_{r=1}^{m}\frac{i\omega\mu_r'}{k_0(1+i\omega\tau_r^{(1)})}+$$
$$+\sum_{r=1}^{n}\frac{i\omega a_r' b_0}{k_0C_0(1+i\omega\tau_r^{(2)})}\cdot\left\{1+\frac{p_0}{\rho_0 b_0}+\sum_{r=1}^{q}\frac{i\omega b_{11}^{(r)}\tau_r^{(4)}}{b_0(1+i\omega\tau_r^{(4)})}\right\}\times$$
$$\times\left\{1+\sum_{r=1}^{s}\frac{i\omega C_r\tau_r^{(3)}}{C_0(1+i\omega\tau_r^{(3)})}\right\}^{-1}\Bigg], \tag{17}$$

with $v_0=\sqrt{k_0/\rho_0}$.

Accordingly, the wave-velocity v and amplitude absorption coefficient a per unit length are given respectively:

$$v=\frac{\omega}{\mathrm{Im}(\beta)}, \tag{18}$$

and

$$a=\mathrm{Re}(\beta). \tag{19}$$

In the special case of $l=m=n=q=s=1$, $\tau=\tau_1=\tau_1^{(1)}=\tau_1^{(2)}=\tau_1^{(3)}=\tau_1^{(4)}$, $b_{11}^{(1)}=C_1=0$, $\lambda_1/k_0\ll1$, $\mu_1/k_0\ll1$, $a_1/k_0\ll1$, $C_1'/C_0\ll1$, $b_{11}/b_0\ll1$ and $b_0/C_0\ll1$, *we* obtain from eq. (17) :

$$\beta^2=-\frac{\omega^2}{v_0^2}\div\left\{1+\frac{i\omega\tau\lambda_1/k_0}{1+i\omega\tau}+\frac{4}{3}\cdot\frac{i\omega\tau\mu_1/k_0}{1+i\omega\tau}+\frac{i\omega\tau a_1 b_0/(k_0C_0)}{1+i\omega\tau}\cdot\left(1+\frac{p_0}{\rho_0 b_0}\right)\right\}=$$
$$=-\frac{(1+i\omega\tau)\cdot\omega^2/v_0^2}{1+i\omega\tau+i\omega\tau\lambda_1/k_0+\frac{4}{3}\cdot i\omega\tau\mu_1/k_0+\left\{i\omega\tau a_1 b_0/(k_0\varepsilon_0)\right\}\left(1+\frac{p_0}{\rho_0 b_0}\right)}=$$
$$=-\frac{(1+i\omega\tau)\cdot\frac{\omega^2}{v_0^2}}{1+i\omega\tau\left(1+\frac{\lambda_1}{k_0}+\frac{4}{3}\cdot\frac{\mu_1}{k_0}+\frac{a_1 b_0}{k_0C_0}+\frac{a_1 p_0}{k_0C_0\rho_0}\right)}. \tag{20}$$

Putting

$$\frac{a_1 b_0}{k_0 C_0}=0, \tag{21}$$

$$E=\frac{\lambda_1}{k_0}+\frac{4}{3}\cdot\frac{\mu_1}{k_0}+\frac{a_1 p_0}{k_0C_0\rho_0}, \tag{22}$$

and $\omega=2\pi f,$ *(23)*

then the expression (20) reduces to:

$$\beta^2=\frac{-\frac{\omega^2}{v_0^2}(1+i\omega\tau)}{1+i\omega\tau(1+E)}=$$
$$=\frac{-\frac{\omega^2}{v_0^2}(1+\omega^2\tau^2+\omega^2\tau^2E-i\omega\tau E)}{[1+\omega^2\tau^2(1+E)^2]}. \tag{24}$$

$$\mathrm{Re}(\beta^2) = -\frac{\frac{\omega^2}{v_0^2}(1+\omega^2\tau^2+\omega^2\tau^2E)}{1+\omega^2\tau^2(1+E)^2} = -\frac{\frac{\omega^2}{v_0^2}\cdot\{1+(2\pi f\tau)^2+(2\pi f\tau)^2\cdot E\}}{1+(2\pi f\tau)^2(1+E)^2}, \quad (25)$$

$$\mathrm{Im}(\beta^2) = \frac{\frac{\omega^2}{v_0^2}\cdot\omega\tau E}{1+\omega^2\tau^2(1+E)^2} = \frac{\frac{\omega^2}{v_0^2}(2\pi f\tau)\cdot E}{1+(2\pi f\tau)^2(1+E)^2}, \quad (26)$$

$$\arg(\beta^2) = \arctan\frac{I_m(\beta^2)}{R_e(\beta^2)}. \quad (27)$$

and

$$\arg(\beta) = \frac{1}{2}\arg(\beta^2) = \frac{1}{2}\arctan\left\{-\frac{2\pi f\tau E}{1+(2\pi f\tau)^2(1+E)}\right\}. \quad (28)$$

We have:

$$\{\mathrm{Re}(\beta^2)\}^2 = \frac{\omega^4}{v_0^4}\cdot\frac{\{1+(2\pi f\tau)^2+(2\pi f\tau)^2\cdot E\}^2}{\{1+(2\pi f\tau)^2(1+E)^2\}^2}, \quad (29)$$

$$\{\mathrm{Im}(\beta^2)\}^2 = \frac{\omega^4}{v_0^4}\cdot\frac{(2\pi f\tau)^2\cdot E^2}{\{1+(2\pi f\tau)^2(1+E)^2\}^2}, \quad (30)$$

$$|\beta| = \sqrt{|\beta^2|} = [\{R_e(\beta^2)\}^2+\{I_m(\beta^2)\}^2]^{\frac{1}{4}}$$
$$= \frac{\omega}{v_0}\cdot\frac{[\{1+(2\pi f\tau)^2+(2\pi f\tau)^2\cdot E\}^2+(2\pi f\tau)^2\cdot E^2]^{\frac{1}{4}}}{\{1+(2\pi f\tau)^2(1+E)^2\}^{\frac{1}{2}}}, \quad (31)$$

and

$$a = \mathrm{Re}(\beta) = |\beta|\cos\{\arg(\beta)\} =$$
$$= \frac{\omega}{v_0}\cdot\frac{[\{1+(2\pi f\tau)^2(1+E)\}+(2\pi f\tau)^2E^2]^{\frac{1}{4}}}{\{1+(2\pi f\tau)^2(1+E)^2\}^{\frac{1}{2}}}\cdot\cos\left[\frac{1}{2}\cdot\frac{2\pi f\tau E}{1+(2\pi f\tau)^2(1+E)}\right], \quad (32)$$

and we obtain the absorption coefficient a devided by the squared frequency f^2:

$$\frac{a}{f^2} = \frac{2\pi}{v_0 f}\cdot\frac{[\{1+(2\pi f\tau)^2(1+E)\}^2+(2\pi f\tau)^2E^2]^{\frac{1}{4}}}{\{1+(2\pi f\tau)^2(1+E)^2\}^{\frac{1}{2}}}\times$$
$$\times\cos\left[\frac{1}{2}\arctan\left\{-\frac{2\pi f\tau E}{1+(2\pi f\tau)^2(1+E)}\right\}\right]. \quad (33)$$

Taking the imaginary part of β, and by means of eqs. (18) and (31), we have :

$$\mathrm{Im}(\beta) = \frac{\omega[\{1+(2\pi f\tau)^2+(2\pi f\tau)^2\cdot E\}^2+(2\pi f\tau)^2\cdot E]^{\frac{1}{4}}}{v_0\{1+(2\pi f\tau)^2(1+E)^2\}^{\frac{1}{2}}}\times$$
$$\times\sin\left[\frac{1}{2}\arctan\left\{-\frac{2\pi f\tau E}{1+(2\pi f\tau)^2(1+E)}\right\}\right], \quad (34)$$

and the sound-velocity becomes :

$$v = \frac{v_0\{1+(2\pi f\tau)^2(1+E)^2\}^{\frac{1}{2}}}{[\{1+(2\pi f\tau)^2(1+E)\}^2+(2\pi f\tau)^2\cdot E^2]^{\frac{1}{4}}\cdot\sin\left[\frac{1}{2}\arctan\left\{-\frac{2\pi f\tau E}{1+(2\pi f\tau)^2(1+E)}\right\}\right]}. \quad (35)$$

Putting $\tau=10^{-7}$ sec, $E=0.1135$ and $v_0=1500$ m/sec in eqs. (33) and (35), we have the curves of velocity versus frequency and of absorption versus frequency

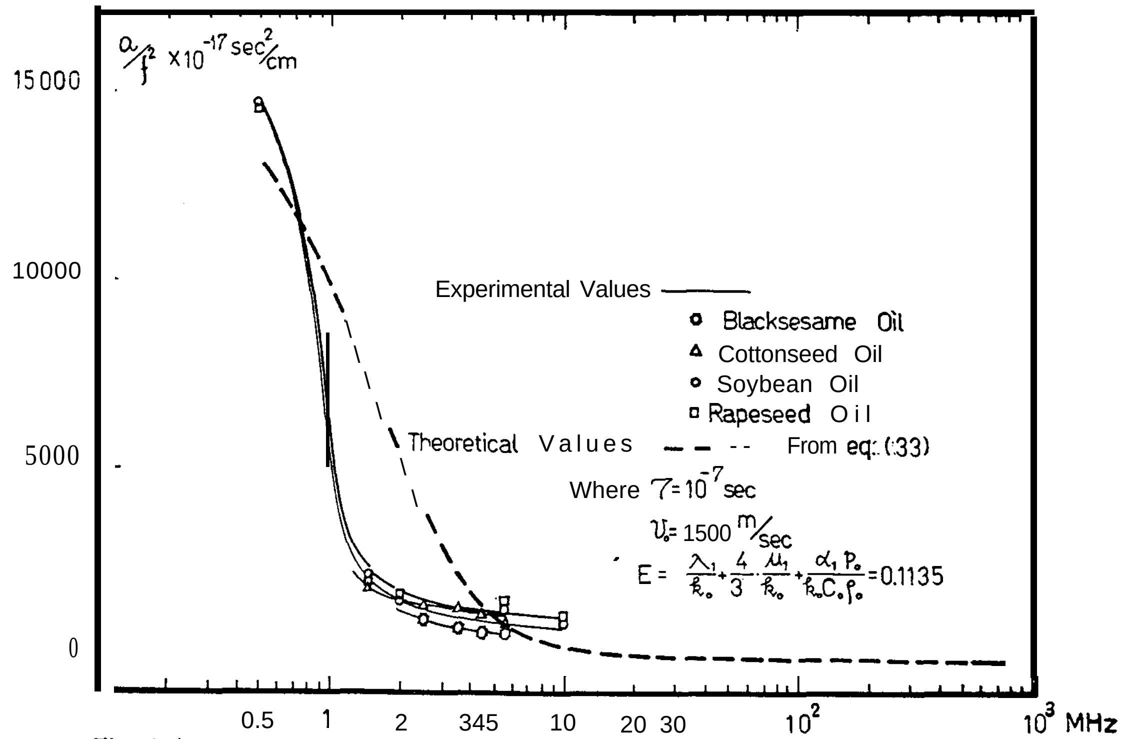

Fig.1.(a) Absorption Vs Frequency.

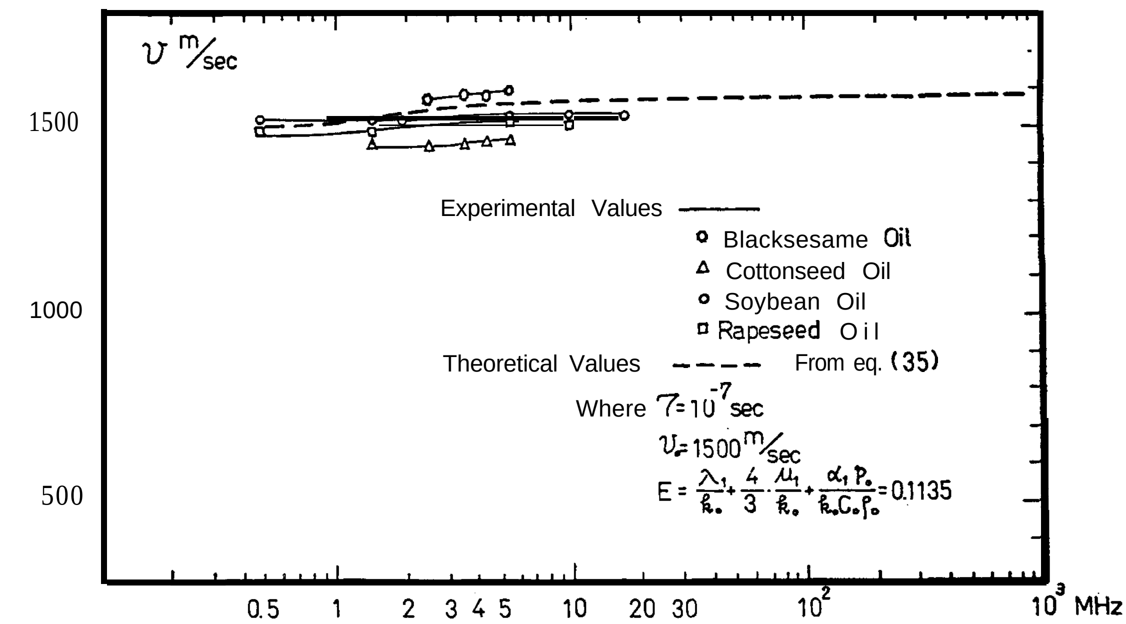

Fig. 1.(b) Velocity Vs. Frequency .

Fig. 1. Absorption Coefficient per Frequency square (α/f^2, s^2/cm) Vs. Frequency (1/s) Theoretical results, and Experimental results (Soybean Oil, Rapeseed Oil and Hyrnndo Oil 90)

as shown in Fig. 1(a) (b). In Fig. 1(b), the absorption increases remarkably when the frequency decreases from 5 MHz to 0.5 MHz.

§3. SAMPLES

Chemical composition and specific gravity

By gas liquid chromatography, developed each oils and the percentage of each fatty acid was calculated from the ratio of area. Also the specific gravity and viscosity of each oils were measured at 30 °C. The result is given in Table 1.

Table 1. Chemical Components, Viscosity and Specific Gravity

Acids \ Oils %	Rapeseed Oil	Soybean Oil	Black Sesame Oil	Cotton seed Oil
Lauric acid	—	—	—	—
Myristic acid	trace	trace	trace	0.7
Palmitic acid	11.3	12.1	11.3	25.3
Palmitoleic acid	trace	trace	trace	0.5
Stearic acid	4.0	3.7	4.9	2.0
Oleic acid	21.8	21.0	34.3	15.2
Linoleic acid	53.4	53.2	43.0	56.7
Linolenic acid	8.0	7.7	2.3	trace
Arachidic acid	—	—	0.5	—
Frucic acid	—	—	1.2	—
Other acid	1.5	2.3	2.5	1.6
Viscosity (cp)	45.6	46.0	51.5	52.0
Specific Gravity	0.915	0.913	0.912	0.916

§4. EXPERIMENTAL PROCEDURE

In the present experiment with ultrasonic frequency of 0.5 MHz-18 MHz, the pulse method was adopted. Here, the author used the "Ultrasonic spectrometer" UAC-5 type, manufactured by Chô-Ompa Kôgyô Kaisha Ltd. Temperature was controled by thermostat "coolnics circulator" CTE-1B and CTR-1B types, manufactured by Komatsu Solidate Co., Ltd.

The Ultrasonic velocity was calculated from the duration of time in which a pulse goes through the liquids between the transducers, and the absorption from the difference between the heights of pulses when the transducers change their distance. For the purpose of diminution of errors due to pulse-form distortions and slowing down of pulse-buildups, short distances (2–5 cm) between the two transducers was used. The distance was read with an accuracy of 10^{-2} mm and the time interval with 10^{-6} sec.

Especially, care was taken to maintain the transmitting and the receiving transducers always kept parallel to each other when they change their distance. And to reduce the error by cabitation the author tries some means to make possible to vanish the bubble in the oil during the experiment. Also the parallax in reading the indicator was avoided. In was observed that the image on the indicator shaked sometimes when the liquid is flowed by external force or thermal convection. So, it is necessary to make measurements when the liquid is perfectly at rest.

§5. RESULTS AND DISCUSSIONS

(1) *Variation of ultrasonic velocity with frequency*

The measured data of each of each oil are shown in Fig. 1. The ultrasonic velocity increases with increasing frequency. The numerical values agree fairly well with the theoretical results obtained in §2.

(2) *Variation of ultrasonic absorption with frequency*

Variation of absorption in three kinds of oils with frequency is shown in Fig. 1. In these oils, the absorption increases remarkably when the frequency decreases from 5 MHz to 0.5 MHz. It is to be noticed that the present data show the same tendency as the theoretical results which were obtained in $2. This kind of behaviour would result from relaxation mechanism of the viscous type. Because of the bulk viscosity, the values of ultrasonic absorption are higher than the Stokes values at low frequency, while the absorption decreases at high frequency because of the relaxation of shear and bulk viscosities.

(3) In the high viscosity oils, the thermal conductivities are small and can be neglected. Because the structures of oils are complex, there are many kinds of relaxation times. This paper counts the relaxation times that are made as the same, so it has only the same tendency compared with the expermiental values. That is, the thermal conductivity can be neglected but the relaxation time needs more kinds. From this paper, it can be seen that the theoretical formula only needs to correct the relaxation time then for high viscosity oils the Maxwellian model can be used.