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Hadronic Matter from QCD Sum Rules

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We will discuss generalization of QCD sum rules to finite temperature and density and apply our method to study changes of hadronic masses (ρ, al, J/Ψ) at finite temperature.

I. INTRODUCTION

With the recent relativistic heavy ion experiments at CERN and at Brookhaven, there is an increasing interest on the properties of hadronic matter under extreme condition. In relation to these experiments, two theoretical aspects are important. These are the theoretical investigation to hadrons at finite temperature and density and a careful analysis of the collision process to understand present heavy ion data. Here, we will use QCD sum rules to study possible changes of hadronic mass at finite temperature. With the sum-rule approach, we can predict hadronic properties in terms of finite-temperature perturbation theory and long-distance nonperturbative physics, which is summarized by quark and gluon condensates. Since we can identify the contribution of each component, we learn about the relative importance of the various quark and gluon condensates in determining bound-state parameters and how thermal effects influence these parameters. Finally, we can directly explore the physical consequences of nonperturbative physics at high temperatures by relating nonzero condensates to observed resonance properties.

II. QCD SUM RULES AT FINITE TEMPERATURE

Let us first review QCD sum rules at zero temperature by considering the time-ordered product of two vector currents such as $J_\mu = \bar{c}\gamma_\mu c$ for charmonium, and look at its vacuum polarization function at Euclidean momentum.

$$T_{\mu\nu}\Pi(Q^2) = i\int d^4x e^{iqx} < 0|T(J_\mu(x)J_\nu(0))|0 >, \quad Q^2 = -q^2. \tag{2.1}$$

Here, $T_{\mu\nu}$ is the tensor structure of the correlator. The function $\Pi(Q^2)$ satisfies the dispersion relation

$$-\frac{d}{dQ^2}\Pi(Q^2) = \frac{1}{\pi}P\int\frac{Im\Pi(s)ds}{(s-q^2)^2} \tag{2.2}$$

The theoretical side of the sum rule is derived from an operator-product expansion of Eq. (2.1):

$$i\int d^4xe^{iqx}T[J(x)J(0)] = C_1\mathbf{1} + \sum_n C_n(Q^2)\mathrm{O_n}, \tag{2.3}$$

The nonperturbative nature of the QCD vacuum enters through the nonvanishing vacuum expectation values of the normal-ordered operators; these expectation values are the condensates. The condensate terms (those not involving the identity operator) are called the power corrections. The operator-product expansion (OPE) incorporates long-distance physics into the vacuum expectation values and short-distance physics into the Wilson coefficients. Corrections due to explicit dependences on the separation scale are expected to be small in QCD sum-rule applications. The phenomenological side of the sum rule follows upon inserting a parametrized model of the spectral density, which is proportional to ImΠ, into the right-hand side of Eq. (2.2). We use the well-established model of the spectral density consisting of a narrow resonance and a smooth continuum with a sharp threshold;

$$Im\Pi(s) = fm_R^2\delta(s-m_R^2) + \theta(s-S_0)Im\Pi(s)_{pert}. \tag{2.4}$$

This side will be called the phenomenological side. The success of the sum rule approach lies in the fact that by taking higher moments of the sum rule or by taking the borel transform of the sum rule, we can find a region where both sides match and hence can get information about resonance parameters.

To formulate QCD sum rules at finite temperature, we have to understand the theoretical side and phenomenological side at finite temperature. Obviously, we have to take the thermal average instead of the vacuum average of the correlator. First thing we have to note is that because of the heat bath, the vacuum is not Lorentz covariant anymore and even for conserved currents the polarization vector will have two independent components, that is transverse or longitudinal to the heat bath.

$$\Pi_{\mu\nu}(k) = -G_tA_{\mu\nu} - G_lk^2B_{\mu\nu}, \tag{2.5}$$

here the tensor structure of A is transverse with respect to the heat bath and B is longitudinal and they are transverse with respect to the four momentum k. Here, we will mainly work with G_l in the $\vec{k}\rightarrow 0$ limit, in which case the two are related through

$$G_t = (k_0^2 - \vec{k}^2)G_l. \tag{2.6}$$

Hence

$$\Pi^{\mu}_{\mu} = -3k^2 G_l, \text{ at } \vec{k} = 0. \tag{2.7}$$

As for the theoretical side, previously we only considered operators that are color singlet and Lorentz covariant in the O.P.E. At finte temperature, we still have to insist on color singlet operators but due to the heat bath, translational invariance is broken and we can not just blindly take thermal average of the O.P.E. that we had before. The normal ordered operators that we neglected because they were not translationally invariant can now contribute at finite temperature. Some examples being,

$$< \bar{\Psi}\gamma_0\Psi >, < \bar{\Psi}\gamma_0 D_0 \Psi >, \quad \text{etc...} \tag{2.8}$$

At finite temperature, we calculate the thermal expectation value of these new operators using a thermal quark basis. At finite density, we can use results from D.I.S. experiments ' .

If there is not any new non-perturbative condensate coming in at finite temperature, then adding all the perturbative thermal condensate into account would be equivalent to calculating each Wilson coefficient using finite temperature propagators. In our calculation, we have used this method for the calculation of the Wislon coefficient of 1. As for the temperature correction to the non-perturbative covariant condensates, we model it consistenly with lattice and mean field results.

III. RESULTS

III-1. ρ meson

This section and the section on charmonium was done in collaboration with R. J. Furnstahl and T. Hatsuda.[2] QCD sum rules work well in this channel for T = 0 case. There is a work by Dosch and Narison[3] who analyzed the sum rule in this channel with temperature dependence only in the thermal Wilson coefficient of 1. They conclude that even around 200 MeV, the effect is small and no sudden change in mass is found contrary to former claims by Bochkarev and Shaposhnikov.[4] Our calculation takes into account both the perturbative temperature effect and the change in the condensate.[2] Our calculation shows that the $<< \bar{q}q >>$ governs the mass shift and other factors are less improtant. This observation is consistent with the picture that chiral symmetry is responsible for the ρ meson properties.

Let us start with the Borel-improved sum rule at T $\neq$ 0 and take thep meson current $(\bar{u}\gamma_\mu u-\bar{d}\gamma_\mu d)/2$ as J_μ In the asymptotic region ($w^2 \to -\infty$), the theoretical side looks as follows,

$$\begin{aligned}(M^2)L_M ReG_l = & \int_0^\infty dw^2 e^{-w^2/M^2}[\theta(w^2-4m_q^2)\rho_g + \delta(w^2)\rho_s] \\ & + \frac{1}{8\pi^2}\left[\frac{a_1}{M^2} + \frac{a_2}{M^4}\right]\end{aligned} \tag{3.9}$$

The first term is the perturbative one-loop contribution, which in our limit of $\vec{k} \to 0$ looks as follows,

$$\rho_g = \tanh\left[\frac{w}{4T}\right]\rho^0$$
$$\rho_s = \int_{4m_q^2}^{\infty} du^2 n_F(u/2T)\rho^0 \qquad (3.10)$$

where $\rho^0 = v(3 - v^2)/16\pi^2$ and $v = (1 - 4m_q^2/w^2)^{1/2}$. The second (third) term corresponds to the gluon (quark) condensates with temperature dependent Wilson coefficients.

$$a_1(T) = a_1'(T)(1 - (T\pi)^2/M^2)$$
$$a_1'(T) = \frac{\pi^2}{3} << \frac{\alpha_s}{\pi}G^2 >> (1 - (T\pi)^2/M^2) \qquad (3.11)$$
$$a_2 = -\frac{448}{81}\pi^3\alpha_s << \bar{q}q >>^2$$

Here, we have assumed $<< E^2 >> = - << B^2 >>$, in accord with the result from lattice calculations.

The phenomenological side of the sum rule, including an ansatz for the spectral function $\rho(w)_{hadron}$, reads

$$\int_0^{\infty} dw^2 e^{-w^2/M^2}[\rho_{pole} + \theta(w^2 - S_0)\rho_{cont} + \delta(w^2)\rho_\pi], \qquad (3.12)$$

where the first term is the ρ-pole contribution, with the form $\rho_{pole} = f m_\rho^2 \delta(w^2 - m_\rho^2)$, and the second term is the phenomenological continuum contribution, which starts from the threshold $\sqrt{S_0}$. The third term, which arises only at finite T, describes the Landau damping caused by the thremal pions. We follow Ref. 4, and approximate the thermal hadronic contribution by that of a free pion gas, which is certainly a valid approximation at low temperature.

Equating the theoretical side to that of the phenomenological side, we get

$$\frac{m_\rho}{M^2} = \left[\int_{4m_q^2}^{S_0} dw^2\left[\frac{w}{m}\right]^2 e^{-w^2/M^2}\rho_g - \frac{1}{8\pi^2}\left[\frac{a_1}{M^2} + \frac{2a_2}{M^4}\right]\right]$$
$$\times\left[\int_{4m_q^2}^{S_0} dw^2 e^{-w^2/M^2}(\rho_g + \rho_s - \rho_\pi) - \frac{1}{8\pi^2}\left[\frac{a_1}{M^2} + \frac{a_2}{M^4}\right]\right]^{-1} \qquad (3.13)$$

Choosing M^2 small enhances the pole contribution while large M^2 suppresses the contribution of higher dimensional operators. Following Dosch and Narison, we have plotted m_ρ as a function of M^2 and sought a region that is insensitive to changes in the threshold parameter. As such a region is found only when $\sqrt{S_0}$ is larger than 1.75 GeV irrespective of T, We fix $S_0 = 4.0$ GeV^2 and search for the minimum of m_ρ (M^2; T) at each temperature. For paramcters at T = 0, we take $\{a_1(0), a_2(0)\} = \{0.05\ \text{GeV}^4, -0.03\ \text{GeV}^6\}$.

In Fig. 1, we show $m_\rho(T)$ for three cases of interests: (i) $a_1(T)$ and $a_2(T)$ are assumed to

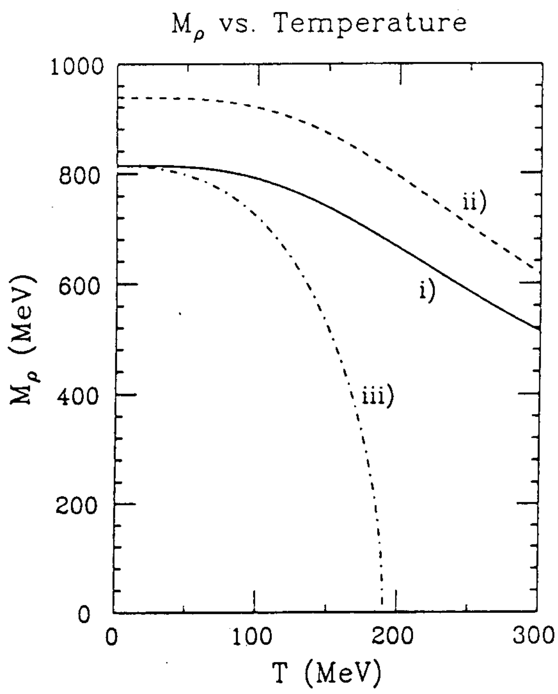

FIG. 1. Sum rule prediction for m_ρ from Ref. 2. $S_0 = 4\ \text{GeV}^2$. (i) $a_1(T)$ and $a_2(T)$ indenpendent of T, (ii) $a_2(T)$ constant but $a_1(T) = 0$, and (iii) $a_1(T)$ constant and $a_2(T) = [1-(T/T_c)^2]^{1/2} a_2(0)$.

be T independent, (ii) $a_2(T) = a_2(0)$ but $a_1(T)$ is assumed to be zero, and (iii) $a_1(T) = a_1(0)$ while $a_2(T)$ varis as

$$\frac{a_2(T)}{a_2(0)} = 1 - \left[\frac{T}{T_c}\right]^2 \tag{3.14}$$

This parameterization is deduced from a simple mean-field assumption

$$<<\bar{q}q>> = <<\bar{q}q>>_{T=0} [1-(T/T_c)^2]^{1/2}, \tag{3.15}$$

From the line (i) in Fig. 1, one sees that the T dependence of the perturbative part is significant only at high temperature, which is consistent with the observation of Dosch and Narison. The line (ii) shows that the gluon condensate is not at all important for the mass shift. The line (iii) shows that $m_\rho(T)$ decreases and vanishes around T_c as $a_2(T)$ tends to zero, which means that $<<\bar{q}q>>$ is essential to forming the ρ meson and so the restoration of the chiral symmetry is correlated with its mass shift. In the real world, the chiral transition is expected to be first order for the light-quark system, based on numerical lattice simulations. In this case, we have to use a more sophisticated parameterization for a_2 to be realistic, but the qualitative feature of our result should not change.

We repeat our analysis of the ρ-meson sum rule for case (i) but now using numerical optimization procedure to extract the spectral parameters. In particular, we minimize the relative difference squared of the theoretical and phenomenological sides of the sum rule with respect to the parameters m_ρ, f, and S_0, as summed over a set of M^2 points. The idea is first to identify a region of M^2 in which we expect the sum rule to be valid, and then to choose the spectral parametes so that the two sides of the sum rule agree most closely in that region. Similar temperature dependence is obtained as in Fig. 1.

A combination of the above methods, namely choose the interval at which neither the continuum threshold or the power correction becomes too large and then by looking at Eq. (3.13) and changing S_0 such that we get the most stable platue in the mass v.s. M^2 plot gives also similar answers.

In summary, our analysis in this section shows that the ρ meson is a bad (good) indicator of the effects of the gluon (quark) condensate at finite temperature.

III-2. a_1 and the mass of the nucleon

In the previous section, we found that for the ρ meson, the perturbative temperature dependence is small and the main quantity that determines the decrease of the ρ meson mass at finite temperature was the temperature dependence of the non-perturbative quark condensate. To see if this behavior is a characteristics of light quark mesons and to study the chiral symmetry restoration effect manifested in meson masses, it is interesting to consider the temperature dependence of the a_1 meson mass.

If the quark condensate were zero, the mass of the a_1 would be the same as that of the ρ. The reason why they are different at zero temperature is because of the broken chiral symmetry. If chiral symmetry restoration is taking place at high temperature, the mass of a_1 should appoach that of the ρ. Moreover, if both masses approach zero near the chiral phase transition, the symmetry restoration will be accompanied by the increase of number of nearly massless particles and would in return modify the properties of hadronic gas near the phase transition.

To calculate the mass of the a_1 meson, it is neccessary to know the temperature dependence of f_π, because the correlator of the axial currents has a pion pole. Rather than taking a temperature dependence of f_π from a model calculation, we will use a form relating f_π and the quark condensate obtained from a constraint of the QCD sum rule approach. This can be obtained by looking at the temperature modification of QCD sum rule derivation of the Goldberg-Treiman relation, whose form was found to be the same at finite temperature from a QCD sum rule point of view.[5]

The finite temperature QCD sum rule in the axial channel has been first looked at by Bochkarev[6] in the lowest order thermal perturbation, and is obtained by looking at the thermal time ordered correlator of axial current

$$\begin{aligned} \Pi_{\mu\nu} &= i\int d^4x e^{iqx} << T[A_\mu(x), A_n u(0)] >> \\ &= -\Pi^T_{\mu\nu} + \Pi^L q_\mu q_\nu \end{aligned} \tag{3.16}$$

where $A\mu = 1/2(\bar{u}\gamma\mu\gamma 5u - \bar{d}\gamma\mu\gamma 5d)$. Here, we will study the sum rule for Π_{00}, which is the same as G_l for the vector current.

Here, we model the phenomenological side as,

$$\begin{aligned} \rho_{phen} = &\frac{f_\pi}{2}\delta(w^2) + f_a m_a^2 \delta(w^2 - m_a^2) + \theta(w^2 - S_0)(1 - 2n_f(w/2T)) \\ &+\delta(w^2)\int_0^\infty du^2 2n_f(u/2T) \end{aligned} \tag{3.17}$$

Where the pion pole is coming from the imaginary part of Π^T. The temperature dependent continuum and scattering is also supposed to include the effect of the $\rho\pi$ intermediate states.

Matching the phenomenological side to the theoretical side with the condensate, the sum rule for a_1 after borel transformation looks as follows,

$$\begin{aligned} 8\pi^2 f_a m_a^2 e^{-m_a^2/M^2} = &8\pi^2 \int_0^{S_0} dw^2 e^{-w^2/M^2}(\rho_g + \rho_s) \\ &+\frac{a_1}{M^2} + \frac{a_2}{M^4} - 4\pi^2 f_\pi^2, \end{aligned} \tag{3.18}$$

where a_1 is the same as in the ρ meson case and

$$a_2 = \frac{704}{81}\pi^3 << \sqrt{\alpha}\bar{q}q >>^2 \tag{3.19}$$

The only difference of this sum rule to that of the ρ meson is the last two terms. Again, by taking the derivative of the above equation and taking the ratio, we get a formula for the mass.

$$\begin{aligned} \frac{m_a^2}{M^2} = &\left[8\pi^2\int_0^{S_0} dw^2 e^{-w^2/M^2}\left[\frac{w}{M}\right]^2(\rho_g + \rho_s) - \frac{a_1}{M^2} - \frac{2a_2}{M^4}\right] \\ &\times\left[8\pi^2\int_0^{S_0} dw^2 e^{-w^2/M^2}(\rho_g + \rho_s) + \frac{a_1}{M^2} + \frac{a_2}{M^4} - 4\pi^2 f_\pi^2, -4\pi^2 f_\pi^2\right]^{-1} \end{aligned} \tag{3.20}$$

We fix the interval of borel mass to be between 0.8 ~ 2.0 GeV^2 and change S_0 to get the least variation for the mass.

In Ref. 6, calculation for the a_1 were done with the condensate and f_π fixed. First, it should be noted that the sudden drop of m_a at around 150 MeV found in Ref. 6 is not found as in the case of the ρ meson. As can be seen in Fig. 2(i), if we keep the condensate and f_π fixed, the

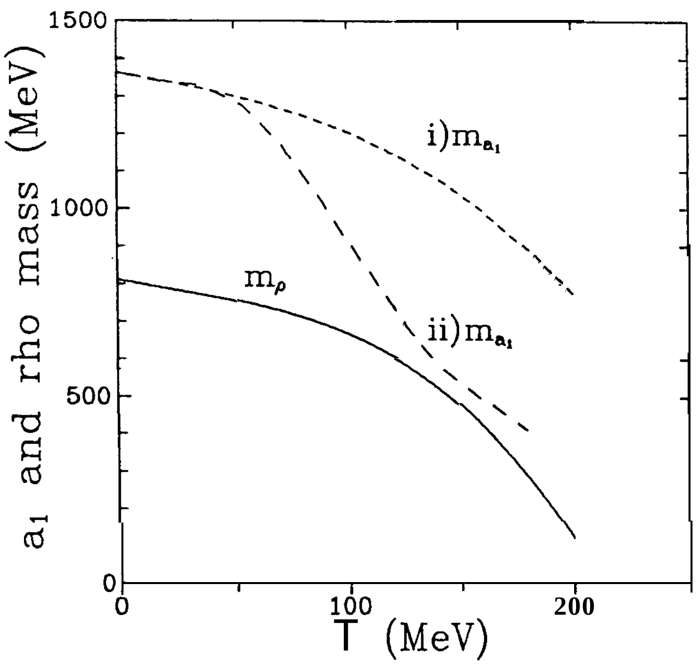

FIG. 2. **Solid line is the result for m_ρ. (i) f_π and $<<\bar{q}q>>$ independent of T, (ii) f_π and $<<\bar{q}q>>$ changed as Eq. (3.23) and Eq. (3.26).**

perturbative thermal correction gives only a smooth decrease in the mass up to 200 MeV and beyond,

To get a realistic temperature dependence, we have to know the temperature dependence of f_π and the condensate. As in the case of the ρ meson, we assume the gluon condensate to be independent of temperature. Hence the temperature dependence of the quark condensate will determine the main features of the temperature dependence of the a_1 mass.

In the sum rule approach, a relation between f_π and other parameters can be obtained by studying the temperature modification to the QCD sum rule derivation of GT relation.

For that purpose, consider the two point function for the baryons, which have been discussed by many authors at zero temperature. Here, we take the thermal expectation value at $\vec{q} = 0$. Then,

$$\Pi(q) = i \int d^4x e^{iqx} << T[\eta_N(x), \eta_N(0)>> = \Pi_1(q^2) + \gamma_0 q_0 \Pi_2(q^2) \tag{3.21}$$

As before, the theoretical side can be calculated using the O.P.E. The lowest order contribution to Π_1 comes from the quark condensate. In a diagrammatic language, this corresponds to one of the quark lines cut to give the condensate and the other two lines forming a loop to give the Wilson coefficient. The temperature correction to this Wilson coefficient can be easily calculated as before using the delta function part times the principal part.

For Π_2, the lowest order contribution comes from the bare two loops. Temperature correction to the real part of this can also be easily calculated by replacing any two of the quark lines by

$$x^\mu << \partial_\mu \Psi(0)\bar{\Psi}(0) >> \tag{3.22}$$

and use the principal part for the other.

This theoretical side is equated to the phenomenological side which we assume to be

$$\Pi(q) = \lambda_n^2 \frac{\gamma_0 q_0 + M_N}{q_0^2 - M_N^2} + \textit{continuum} \tag{3.23}$$

The sum rules for Π_1 and Π_2 are obtained by equating the phenomenological side to the theoretical side. Here, we will neglect the contribution from the continuum. This approximation will become questionable near the phase transition region where the effect of continuum will become greater. After taking the bore1 transformation, the two sum rules will look as follows,

$$2aM^4 Y\,(T) = 2(2\pi)^4 \lambda_N^2 M_N \exp(-M_N^2/M^2) \tag{3.24}$$

$$M^6 X(T) = 2(2\pi)^4 \lambda_N^2 \exp(-M_N^2/M^2). \tag{3.25}$$

where

$$\begin{aligned} a &= -(2\pi)^2 << \bar{q}q >> \\ Y(T) &= 1 - \frac{56}{15}\left(\frac{T\pi}{M}\right)^4 \\ X(T) &= 1 - \frac{56}{3}\left(\frac{T\pi}{M}\right)^4 + \pi^2 << \frac{\alpha}{\pi}G^2 >> /M^4 + \frac{4}{3}a^2/M^6. \end{aligned} \tag{3.26}$$

Here we have taken the proton current to be

$$\eta_N(x) = \epsilon_{abc}(u^{a^T}(x) C\gamma_\mu u^b(x))\gamma_5\gamma^\mu d^c(x). \tag{3.27}$$

A third sum rule is obtained by considering the baryon correlator not in the vacuum but with one external pion. Then the theoretical side will select only the operator&q and the Wilson coefficient will also be the same as above and to lowest order, the zero temperature result will be corrected by a multiplicative factor Y(T). The phenomenological side can be obtained by using the effective pion nucleon interaction Lagrangian $L = g_{\pi NN}\bar{N}i\gamma_5(\tau\cdot\pi)N$. Then after the borel transformation, the sum rule will be

$$\frac{1}{\sqrt{2}}\frac{1}{(2\pi)^2}M^4\frac{f_\pi m_\pi^2}{2m_q}Y(T) = g_{\pi NN}\lambda_N^2 \exp(-M_N^2/M^2). \tag{3.28}$$

A fourth sum rule is obtained from considering the baryon pion three point function. Here there are two momenta involved. The nucleon momenta and the pion momenta. The sum rule that people use is the one obtained by looking at the pion pole and making a borel transformation with respect to the nucleon momenta. It is easy to show that the same temperature correction exist here in the theoretical side such that with the same effective Lagangian as above, we have a fourth sum rule,

$$M^2\frac{f_\pi^2}{(2\pi)^2}Y(T) = \lambda_N^2 \exp(-M_N^2/M^2)\frac{M_N}{M^4}g_{\pi NN}\frac{f_\pi}{\sqrt{2}}. \tag{3.29}$$

The four sum rules we have discussed all have the bore1 parameter M^2. It is obvious that this parameter should be the same in the first second and fourth sum rule. It is argued that the M^2 in the third sum rule, which is also the borel mass of the baryon momentum, will be similar to the others. Hence the four sum rules have λ_N, M_N, $g_{\pi NN}$ and M^2. So we can at least know the ratios of the above parameters.

From Eq. (3.24) and Eq. (3.28), we get the GT relation. Note that the temperature dependence cancell out and there is no temperature modification,

$$g_{\pi NN} = \frac{\sqrt{2}M_N}{f_\pi}. \tag{3.30}$$

From Eq. (3.25) and Eq.(3.29), we get another relation for $g_{\pi NN}$

$$g_{\pi NN} = 2\sqrt{2}(2\pi)^2\frac{f_\pi}{M_N}\frac{Y(T)}{X(T)}. \tag{3.31}$$

From the two relation for $g_{\pi NN}$, we get

$$f_\pi^2 = M_N^2\frac{X(T)}{2(2\pi)^2 Y(T)}. \tag{3.32}$$

We observe here that the non-perturbative temperature dependence of f_π is related to the mass of the baryon with a perturbative temperature correction, so that if the mass of the nucleon changes because of some non-perturbative reasons, for example chiral symmetry restoration etc., then f_π should scale similarly with a small perturbative temperature correction.

To calculate the temperature dependence of f_π, we first have to understand how the nucleon mass changes with temperature. From Eq. (3.24) and Eq. (3.25), we get the sum rule for the nucleon,

$$M_N(M^2) = \frac{2aY(T)}{M^2X(T)} \tag{3.33}$$

where for simplicity, we have neglected the gluon condensate in X(T). Numerically, its contribution is small. We will look at this nucleon sum rule to get the temperature dependence of f_π and then use this result to get the temperature dependence of the a_1 mass.

The nucleon sum rule can be analyzed in the usual way by looking at the stable platue with respect to variation of M^2. The form of Eq. (3.33) is especially good for this kind of analysis because it indeed has an extremum with respect to variation of M^2. Using this fact, we make a variation of Eq. (3.33) with respect to M^2 and find the value of M_{min}^2 at which the nucleon mass is an extemum. To lowest order in T,

$$M_{min}^2 = \left(\frac{8}{3}a^2 + \frac{56}{15}8(T\pi)^4\left(\frac{8}{3}a^2\right)^{1/3}\right)^{1/3} \tag{334}$$

Substituting this value of M_{min}^2 to Eq. (3.33) and Eq. (3.32), we get the temperature dependence of M_N and f_π. Here we assume the same mean field variation of the quark condensate as before.

As can be seen from Fig. 3 and Fig. 4, these values go to zero as a or the quark condensate does. From Eq. (3.30), we can also study how $g_{\pi NN}$ changes at finite temperature. The result

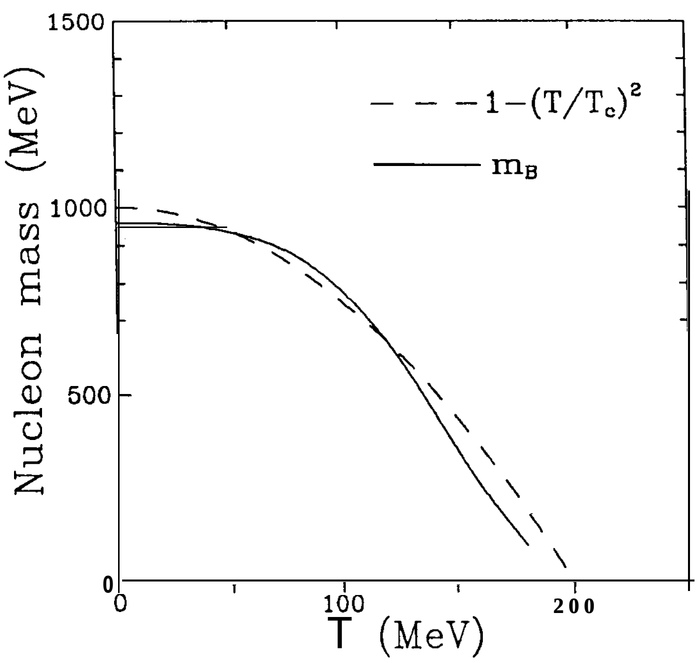

FIG. 3. Solid line is the mass of the nucleon as a function of temperature. Dashed line is the normalized plot of temperature dependence of $<<\bar{q}q>>_T^2 = [1 (T/T_c)^2]$ and T_c = 200 MeV.

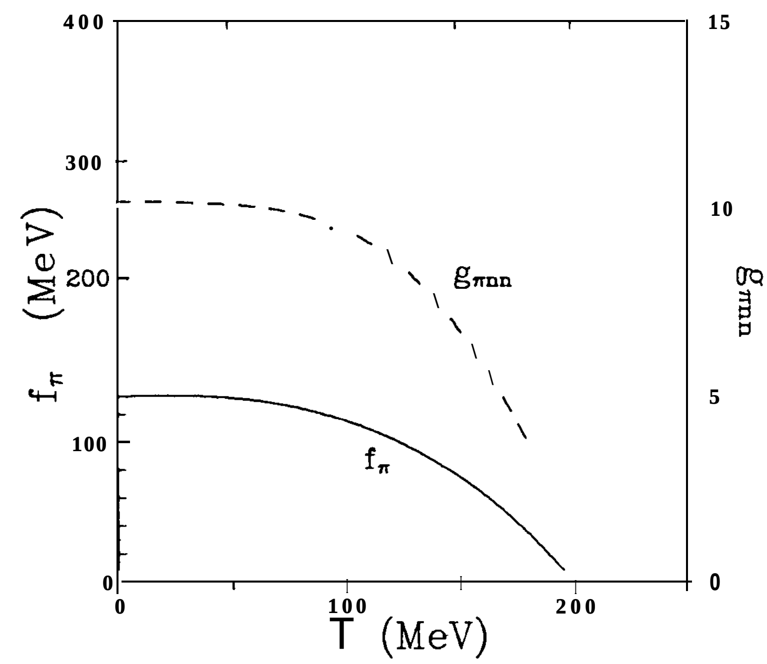

FIG. 4. Temperature dependence of f_π and $g_{\pi nn}$.

is shown in Fig. 4 and seems consistent with other results using chiral models.

We can use the temperature dependence of f_π to calculate the temperature dependence of the axial meson using Eq. (3.20). We use similar method as before. Namely, we take the interval of M^2 for which the zero temperature sum rule was reliable and for this interval find the value of the continuum threshold that has the most stable value of m_a from Eq. (3.20). It is also worth mentioning that in these interval of bore1 mass, both the continuum and power corrections are under control. As can be seen from Fig. 2, the mass of a_1 approaches that of the ρ meson and both fall to zero. It is difficult to identify the changes as exactly the same as the temperature dependence of the quark condensate or any power of it, however, it is certainly induced by it. There is a derivation of the Weinberg relation $m_a = \sqrt{2}m_\rho$ in the sum rule approach, which however needs the assumptions that the continuum of the ρ be the same as that of the a_1 and the KSFR relation. Even at finite temperature if we still assume $S_\rho = S_a$ and KSFR, the temperature corrections are such that the Weinberg relations should still hold. However, there is no proof for or against this and numerically, m_ρ/m_a does seem to have a temperature dependence.

Anyhow, it seems that for light quark system, the earlier conclusion that the temperature dependence of quark condensate dominantly determines the temperature dependence of ρ meson mass also applies to a_1 meson mass, f_π and M_N.

Other current algebra relations such as the Gell-Mann Oakes Renner relation can be shown to have no temperature modification to lowest order.

III-3. J/Ψ

Zero temperature sum rules for charmonium have been extensively studied by Reinders et al. using the moment method and by Bertlmann[7] using the Borel transform method.

In heavy quark system, the dimension four condensates are most important. In addition, the contribution from the heavy-quark condensate vanishes to leading order in the heavy-quark mass expansion. Thus, the charmonium spectrum is primarily determined in the sum-rule approach only by perturbation theory and $<< G^2 >>$. Here, to first approximation, we only consider the effect of lowest order thermal perturbation effect. With this assumption, the theoretical side reads

$$(M^2)L_M Re\Pi_l = \int_0^\infty dw^2 e^{-w^2/M^2}[\theta(w^2 - 4m_c^2)\eta_g + \delta(w^2)\eta_s] + \frac{1}{2}e^{-4m_c^2/M^2} A(M^2)[\alpha_s a(M^2) + \phi b(M^2)] \tag{3.35}$$

where

$$\phi = \frac{4\pi^2}{9(4m_c^2)^2} << \frac{\alpha_s}{\pi} GG >> \tag{3.36}$$

and $A(M^2)$, $a(M^2)$ and $b(M^2)$ are given in Ref. 7

The phenomenological side of the sum rule will be modeled with the scattering contribution from the thermal D mesons, which are analogous to the pion contribution in the ρ meson sum rule.

$$\int_0^\infty e^{-w^2/M^2}[\rho_{pole} + \theta(w^2 - S_0)\rho_g' + \delta(w^2)(\rho_D' + \rho_{D_s}')] \tag{3.37}$$

where $\rho_{pole} = fm^2_{J/\Psi}\delta(w^2 - m^2_{J/\Psi})$, ρ_g' is the same as ρ_g with $\sqrt{S_0}/2$ replacing m_q and a multiplicative factor of $(1 + \alpha_s)$, and ρ_D' and ρ_{Ds}', are the D-meson contributions, with a multiplicative factor $(1 + \alpha_s/\pi)$.

After equating the theoretical and phenomenological sides, the logarithmic derivative with respect to $1/M^2$ gives us the following expression for the J/Ψ mass:

$$
\begin{aligned}
m^2_{J/\Psi} = \; & \Big[\int_{4m_c^2}^{\infty} dw^2 w^2 e^{-w^2/M^2}\rho_g - \int_{S_0}^{\infty} dw^2 w^2 e^{-w^2/M^2}\rho_g' \\
& +2m_c^2 e^{-4m_c^2/M^2} A(\alpha_s a + \phi b)\left[1 - \frac{A'}{A} - \frac{\alpha_s a' + \phi b'}{\alpha_s a + \phi b}\right]\Big] \\
& \times \Big[\int_{4m_c^2}^{\infty} dw^2 e^{-w^2/M^2}\rho_g - \int_{S_0}^{\infty} dw^2 e^{-w^2/M^2}(\rho_g' + \rho_s - \rho_D' - \rho_{D_s}') \\
& +\frac{1}{2}e^{-4m_c^2/M^2} A(\alpha_s a + \phi b)\Big]^{-1}.
\end{aligned}
\tag{3.38}
$$

In this ratio method, we have to make sure that we are studying the range of M^2 where the power correction is small. In our work, we keep the gluonic power correction less than 30%. The assumption is that neglected power corrections are of the order of the square of the power corrections that are kept, and so are small. To determine the resonance mass and continuum threshold, we search for the range of M^2 for which the power correction is smaller than 30% using the ratio method and then minimize the relative difference of the theoretical side and the phenomenological side in this range. Using the new parameters, a new range of M^2 is determined and the process is iterated until the range is self-consistent.

We use the following parameter set:

$$
\begin{aligned}
& m_c = 1.42GeV \text{ or } m = 1.28GeV, \\
& \alpha_s = 0.27,\ \phi = 1.23 \times 10^{-3}.
\end{aligned}
\tag{3.39}
$$

We follow the prescription of Bertlmann in choosing the renormalization point to be at the physical mass $m_c^2 = m^2(p^2 = m^2)$. Applying the criteria discussed above, the M^2 range is from about 0.7 to 1.4 GeV^2.

In Fig. 5, we show how the resonance parameters change as a function of the temperature, assuming a constant value for the condensate to isolate the perturbative thermal effects. Only small changes in the parameters are predicted until T ~ 100 MeV, at which point the resonance and the continuum threshold decrease suddenly. The exact position of this decrease depends somewhat on the parameters used and will shift if we include higher-order corrections and introduce the temperature dependence of the condensate. However, the sudden change in the threshold seems to indicate that the sum rule breaks down at approximately this temperature and that the bound state may disappear.

The sudden change is caused by the scattering term ρ_S, wich incorporates the effects of the charm-quark thermal bath. It is surprising, at first, that this thermal bath should have an influence on the resonance properties, because T ~ 100 MeV < < m_c and so the thermal factor n_F is quite small. However, in the sum rule all terms except for ρ_S scales like $n_F = e^{-m_c/T}$. Thus, at a given temperature, the scattering contribution overwhelms the other terms for M^2 less than

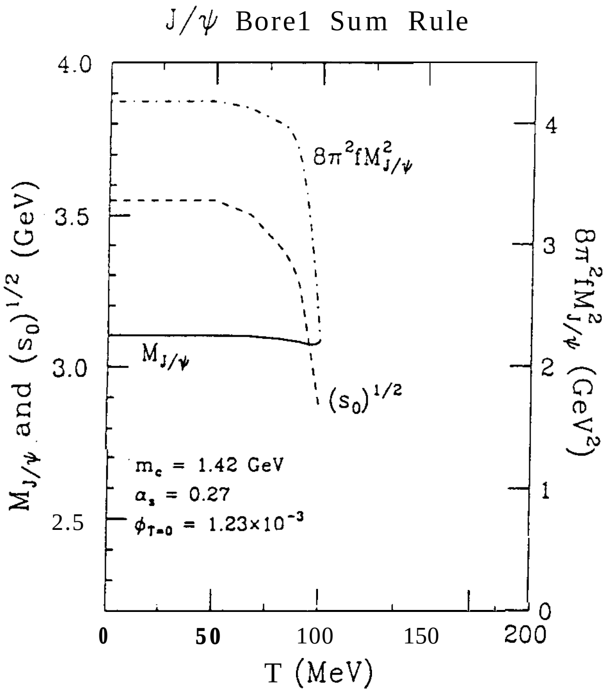

FIG. 5. Resonance parameters of the J/Ψ as a function of temperature, by optimizing the Bore1 sum rule.

a characteristic value. As the temperature is increased, this value moves into the region where the power correction is less than 30%, at which point the scattering term dominates the sum rule and precipitates the sharp changes in the spectrum shown in Fig. 5.

In Ref. 2 the reason for this sudden changes is explained in terms of almost massless pole coming from the scattering term, which however, does not stand out in light quark mass system.

111-4. Summary and Discussion

Let us summarize our results so far, for light quark system including the nucleon, the chiral condensate in the vacuum seems to be the dominant factor in determining the temperature dependence of the changes of masses at finite temperature.

For heavy quark system, the gluon condensate seems to be responsible for forming a bound state. However, the perturbative effect dominate the non-perturbative effect above certain temperature, makeing a perturbative treatment of the bound state justifiable.

Of course there are limitations to the sum rule approach at finite temperature. At zero temperature, we have only one scale M^2 in the sum rule in addition to Λ_{QCD}. At finite temperature, we have one additional scale T. The relation between Λ_{QCD} and T is a question on the validity of how high T should be for the thermal perturbation to make sense in QCD, we will not discuss this question here. Even so, we have two scales M^2 and T. When $T < M$, the low temperature expansion is OK. Our formula for the mass of the nucleon and f_π corresponds to this case. This implies that our formula should not be trusted near the phase transition region T ~ 200 MeV where higher order correction in (T/M) is believed to be important. At high temperature, for certain order of a, the temperature correction to all orders in T/M should be added, which corresponds to using the usual finite temperature propagator instead of the simple one. The question as to which method is better for describing the phenomena at the phase transition region is an open question.

The phenomenological side itself should be drastically modified near the phase transition region because many particles become massless and hence scattering from other particles should also be included. Then, the question of how to model the phenomenlogical side becomes very difficult. Overall, our sum rule approach should not be trusted near the phase transition region. However, the behavior below T ~ 150 MeV already shows the general trend for temperature dependence of hadronic parameters and our conclusion should remain valid.

Therefore, when analysing RHIC data, one has to take into account changes of meson masses depending on the species.

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