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Cut-Off Model of the Quark-Gluon Plasma

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The phenomenological model for the quark-gluon plasma equation of state is discussed. We analyse data of Monte-Carlo simulations in lattice gauge theory for the thermodynamical functions and heavy-quark potential in the temperature region above the deconfinement transition.

I. INTRODUCTION

Monte-Carlo (MC) simulation in lattice gauge theory is the unique quantative method to study non-perturbative effects in quantum chromodynamics (QCD) at finite temperatures (see, for example, reviews[1]). MC data for thermodynamical functions can be used to probe the validity of phenomenological models for the Quark-Gluon Plasma (QGP) equation of state (EOS). It was observed that the pressure of the QGP strongly deviated from the ideal gas one[2,3] and in the discussion of this problem simple non-perturbative model of the QGP was suggested.[4-8] We call it (momentum) cut-off model. The high momentum partons are assumed to be weakly interacting, and consequently can be treatead in a perturbative way. The low-momentum partons are supposed to interact strongly and their density is substantially depleted because of the formation of massive hadron-like modes. The idea of existence of such hadron-like modes in the QGP has been considered by several authors.'

The simplest way to formulate a model based on the presented idea is to assume that there are no partons in the system with momenta smaller than a critical value K, while the partons with momenta greater than K behave as in perturbative QGP and their momentum distributions are given by the Bose and Fermi distributions, respectively for gluons and quarks.

In the cut-off model we treat quarks and gluons in the same way (see Refs. 6 and 7), although, it is not quite clear at present because most of the MC calculations deal with pure gluodynamics. It was shown in Ref. 7 that the modification of the quark momentum spectrum becomes very important in the case of finite baryonic densities, which has not been studied yet in the lattice QCD.

The aim of our consideration is to show that the cut-off model can give a good description of lattice MC data for the gluon plasma (GP). We analyze the MC data of the GP for the thermodynamical functions and heavy-quark potential. Model parameters will be fixed in the calculations of the thermodynamical functions, therefore, the calculation of heavy-quark potential contains no additional free parameter.

II. CUT-OFF MODEL FOR THE THERMODYNAMICAL FUNCTIONS OF THE GP

Following[2-8] we choose the gluon distribution function of GP in the form

$$f(k) = \frac{d_g}{(2\pi)^3} \frac{\theta(k-K)}{\exp(k/T)-1}, \tag{1}$$

where $d_g = 2(N_c^2-1)$ is the degeneracy factor for $SU(N_c)$-gluons, T is the temperature of the GP and $k = |\vec{k}|$. With a general form of the gluon dispersion relation w(k) one has for the GP pressure

$$p(T) = -T\frac{d_g}{(2\pi)^3}\int d^3k \ln\left[1-\exp\left(-\frac{\omega(k)}{T}\right)\right]. \tag{2}$$

The energy density is obtained from Eq. (2) by the well-known thermodynamical relation

$$\epsilon(T) = T\frac{dp}{dT} - p. \tag{3}$$

All phenomenological modifications of the EOS should be introduced in the pressure function. The energy density is obtained then from general thermodynamical relation (3), and there is a guarantee of the thermodynamical selfconsistency of the model EOS (see Ref. 3). Cut-off model (1) means that

$$\omega(k) = k, \quad k > K; \qquad \omega(k) = \infty, \quad k < K. \tag{4}$$

Another possible modification of w(k) to suppress low-momentum gluons is the introduction of a non-zero (non-perturbative) gluon mass M_g (see Ref. 10), so that $w(k) = (k^2 + M_g^2)^{1/2}$.

Assuming that cut-off parameter K in Eq. (1) is a function of T we have from (2,3)

$$p(T) = \frac{1}{3}\int d^3k k f(k), \tag{5}$$

$$\epsilon(T) = \int d^3k k f(k) + \frac{d_g}{2\pi^2}(TK)^2 \ln[1-\exp(-K/T)]\frac{dK}{dT}. \tag{6}$$

After introduction of non-perturbative cut-off effect we treat the GP in a perturbative way. The coupling constant g^2 of the modified perturbative expansions can be represented in the form:[11]

$$\frac{g^2}{4\pi} = \frac{6\pi}{11/2} N_c \ln(M^2/\Lambda^2), \tag{7}$$

where A is the QCD scale parameter, and the mass parameter M is defined as[11]

$$M^2 = \frac{4}{3} < \vec{k}^2 >, \tag{8}$$

where $< \vec{k}^2 >$ is the thermal average of the gluon squared momentum. It is important that within the cut-off model the coupling constant Eq. (7) is small at all temperatures in the deconfined phase and modified perturbative expansion seems applicable everywhere in the GP.

The cut-off model has been used to describe very precise SU(2) data for the pressure and energy density.' Since the deconfinement phase transition in the SU(2) GP is of the second order, it has been assumed that the cut-off parameter K varies with the temperature as $(T-T_c)^\alpha$ where T_c is the phase transition temperature. Fitting the data, the following relation has been found[5]

$$K/T_c = 2.9[(T - T_c)/T_c]^{-0.30}. \tag{9}$$

In the bag model EOS (see, e.g.,[12]) one takes into account the difference between the energy densities of the perturbative and non-perturbative vacuum states, but uses simple dispersion relation w(k) = k. It results to

$$p(T) = \frac{\sigma}{3} T^4 - B, \quad \epsilon(T) = \sigma T^4 + B, \tag{10}$$

where $\sigma = d_g\pi^2/30$ is the Stefan-Boltzmarm constant for $SU(N_c)$ gluons and B is the bag constant ("vacuum pressure"). The analysis shows however that Eq. (10) is in a qualitative disagreement with MC data (see also Ref. 3). Besides, in such a model gluon momentum distribution function remains the ideal Bose-Einstein one, but this in contradiction with MC data for heavy-quark potential (see below).

III. HEAVY-QUARK POTENTIAL

The potential V acting between a heavy quark and antiquark, which is calculated on the lattice, is a combination of the singlet V_1 and adjoint V_8 potentials. For the $SU(N_c)$ group this potential is expressed as[13,14]

$$\exp\left[-\frac{V(r,T)}{T}\right] = \frac{1}{N_c} \exp\left[-\frac{V_1(r,T)}{T}\right] + \frac{N_c^2 - 1}{N_c^2} \exp\left[-\frac{V_8(r,T)}{T}\right], \tag{11}$$

where r is the distance between the quark and the antiquark. In the perturbative limit the singlet potential is[14]

$$V_1(r,T) = -\frac{g^2}{4\pi}\frac{(N_c^2-1)}{2N_c}\frac{\exp(-m_D r)}{r} = -(N_c^2-1)V_8(r,T), \tag{12}$$

where m_D is the Debye screening mass. Assuming that $|T/V_{1,8}| >> 1$, which is confirmed by the lattice data[15,16] for the range of r and T discussed below, one finds from Eqs. (11,12)[14]

$$-\frac{V(r,T)}{T} \cong \left(\frac{g^2}{4\pi}\right)^2 \frac{(N_c^2-1)}{8N_c^2}\frac{\exp(-2m_D r)}{(rT)^2}. \tag{13}$$

The Debye screening mass m_D can be found from the kinetic-theory approach[17] (see also Ref. 18). For the gluon distribution (1) it has the following form[8]

$$m_D^2 = \frac{g^2 N_c T^2}{\pi^2} I(K/T), \tag{14}$$

where

$$I(a) = \int_a^\infty \frac{dx x^2 e^x}{(e^x-1)^2} - \frac{a^2}{e^a-1}. \tag{15}$$

The coupling constant $g^2(T)$ is defined by Eq. (7) with mass parameter M (8) equals[8]

$$M^2 = \frac{4}{3}T^2 \frac{\int_{K/T}^\infty dx x^4 (e^x-1)^{-1}}{\int_{K/T}^\infty dx x^2 (e^x-1)^{-1}} \tag{16}$$

To compare the continuum potential (13) with the lattice data one has to relate the continuum QCD scale parameter Λ to the lattice scale parameter Λ_L. It has been proved in Ref. 19 that Λ = 57.5 Λ_L for N_c = 2. If the temperature and the scale parameter Λ are measured in units of the critical temperature T_c, which equals 37.9 Λ_L,[5] we do not need to know the actual value of Λ_L.

In Fig. 1 we present the interquark potential in SU(2) GP for two values of temperature. The points are the MC results[15] extrapolated to an infinite continuum system. The solid lines correspond to the cut-off model,[8] while the dashed lines represent the standard perturbative calculations (K = 0). It is seen that the cut-off model describes the data very well, and there is a (grave) contradiction with MC data (at T near T_c) in standard perturbative calculations. A similar problem one faces in the bag model, where the calculation of the heavy-quark potential is the same as in standard perturbative approach.

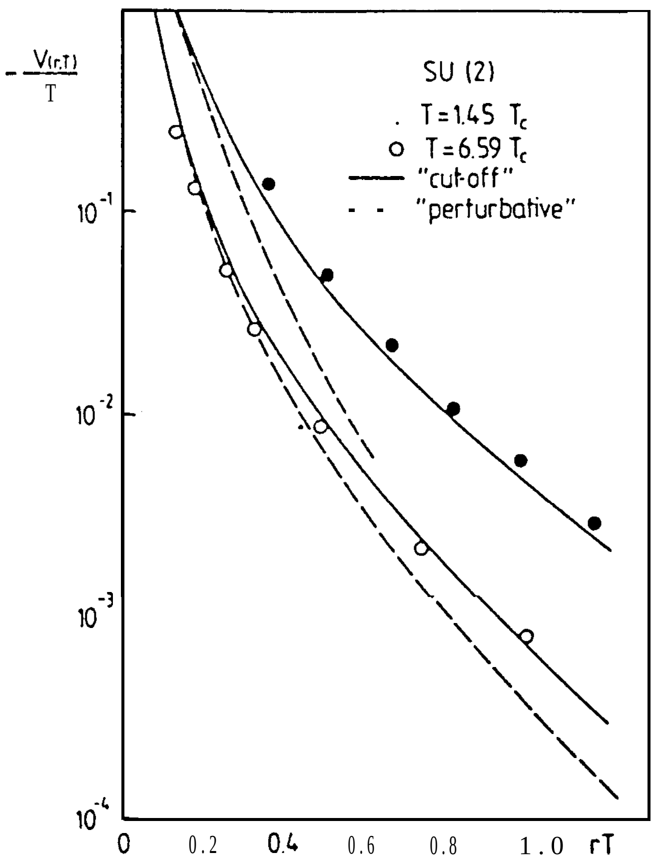

FIG. 1. The interquark potential for the SU(2) GP. The points are MC data." The solid lines correspond to the cut-off model,[8] while the dashed ones represent the standard perturbative calculations (K = 0).

IV. CONCLUSION

Improved data for the pressure and energy density of the SU(3) GP have been recently presented in Ref. 20. The non-perturbative relation between the lattice coupling constant and lattice spacing (see Ref. 21) has been used to obtain the thermodynamical functions of the GI? The cut-off model description of these data as well as data[16] for heavy-quark potential in SU(3) GP is now in progress.

In conclusion, we have shown that MC data for the thermodynamical functions and heavy-quark potential in the GP can be described in the model with modified (non-perturbative) gluon dispersion relation which corresponds to the suppresion of gluons with momenta smaller than some critical value K. The investigation of both the physical origin and possible experimental signatures of such a model desrves, in our opinion, further efforts.

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