Numerical Solution for the Energy Density of a Static Spherically Symmetric Gravitational Field

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A numerical solution for the energy density of the gravitational field of a static spherically symmetric perfect fluid neutron star in Cartesian coordinates is calculated with two equations of state. The results show that the energy densities of the gravitational field of neutron stars can be nearly up to the saturation density of nuclear matter.

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I. INTRODUCTION

Because of the strong gravitational field, relativistic stars such as neutron stars, which are ideal astrophysical laboratories for testing theories of dense matter physics and strong gravitational fields, must be studied in the framework of general relativity. For strong gravitational field research, the energy of a gravitational field is one of the important subjects needing serious study. In order to get an appropriate definition of the energy of a gravitational field, considerable effort has been expended [1, 2]. Among these efforts, the quasilocal idea and the quasilocal energy of gravitational and matter fields have been widely used and accepted [1, 3, 4]. Another effort made use of pseudotensor methods, which are coordinate-dependent. Several decades ago investigators had made lots of attempts, different expressions for the energy of a gravitational field were given, such as the Einstein-Tolman expression, the Landau-Lifshitz expression, the Møller expression etc. [2]. As we know based on the Einstein-Tolman expression, if the energy density of the gravitational field of a spherically symmetric star is calculated in spherical coordinates we will get an irrational result, that is, the energy density of the gravitational field is negative and the total energy is infinite. Similar situations will happen if we calculate the energy of cylindrical gravitational waves in cylindrical coordinates. Rosen *et al.* [5] calculated the energy of cylindrical gravitational waves in Cartesian coordinates. They got a positive and rational energy density. Stimulated by Rosen's work, we calculate the energy density for a spherical symmetric gravitational field in Cartesian coordinates and also get a positive and rational analytical expression for the energy density and a rational total energy of the system [6]. In this paper, the numerical result for the energy of the gravitational field of the neutron stars with two different equations of state (EOS) will be given. By the way, using quasi-local approaches to deal with the energy-momentum has more advantages than using the pseudo-tensor methods. For a detailed review of the quasi-local approaches please see [7].

Here we adopt the metric signature as $(+---)$, and $G = c = 1$.

II. THE ENERGY DENSITY OF A STATIC SPHERICALLY SYMMETRIC GRAVITATIONAL FIELD IN CARTESIAN COORDINATES

According to the conservation law of energy one has $T^{\nu}_{\mu;\nu} = 0$, where $T_{\mu\nu}$ is the energy-momentum tensor of matter. In view of the symmetry of $T_{\mu\nu}$, we have

$$T^{\nu}_{\mu;\nu} = (-g)^{-\frac{1}{2}}(T^{\nu}_{\mu}\sqrt{-g})_{,\nu} - \frac{1}{2}g_{\alpha\beta,\mu}T^{\alpha\beta}\,, \tag{1}$$

Suppose $(-g)^{-\frac{1}{2}}(t^{\nu}_{\mu}\sqrt{-g})_{,\nu} = -\frac{1}{2}g_{\alpha\beta,\mu}T^{\alpha\beta}$, then

$$[(T^{\nu}_{\mu} + t^{\nu}_{\mu})\sqrt{-g}]_{,\nu} = 0\,. \tag{2}$$

Eq. (2) is just the general differential form of the conservation law, in which $\sqrt{-g}\,t^{\nu}_{\mu}$ is a pseudo-tensor. From Eq. (2), one will naturally consider $\sqrt{-g}\,t^{\nu}_{\mu}$ as a part of the contribution of the gravitational field to the total energy-momentum tensor.

Define the total energy-momentum tensor as

$$\tau^{\nu}_{\mu}\sqrt{-g} = t^{\nu}_{\mu}\sqrt{-g} + T^{\nu}_{\mu}\sqrt{-g}\,, \tag{3}$$

which can be expressed as follows [8]

$$\tau^{\nu}_{\mu}\sqrt{-g} = \frac{1}{16\pi}H^{\nu\sigma}_{\mu,\sigma} = \frac{1}{16\pi}\{(-g)^{-\frac{1}{2}}g_{\mu\lambda}[-g(g^{\nu\lambda}g^{\sigma\gamma} - g^{\sigma\lambda}g^{\nu\gamma})]_{,\gamma}\}_{,\sigma}\,. \tag{4}$$

From Eq. (4), one can see that the total energy-momentum tensor can be expressed by only the metric tensor. Then according to Eq. (3), one can get the expression for the energy-momentum pseudo-tensor of the gravitational field as

$$\theta^{\nu}_{\mu} = \sqrt{-g}\,t^{\nu}_{\mu} = \frac{1}{16\pi}H^{\nu\sigma}_{\mu,\sigma} - \sqrt{-g}\,T^{\nu}_{\mu}\,. \tag{5}$$

This is just the Einstein-Tolman energy-momentum pseudo-tensor. For the spherical symmetric stars, if the matter can be looked on as a perfect fluid, then the energy-momentum tensor of the matter can be denoted as

$$T^{\nu}_{\mu} = (p+\rho)u^{\nu}u_{\mu} - p\delta^{\nu}_{\mu}\,, \tag{6}$$

where p is pressure, ρ is density, and u^{ν} is the four-velocity.

By using Eqs. (4) $\sim$ (6) to calculate the energy of the gravitational field in spherical coordinates, a negative energy density will be obtained. But if the energy of the gravitational field is calculated in Cartesian coordinates, the result will be rational [6]. The energy density is the most important component of the energy-momentum tensor; in this work, we only calculate and discuss the energy density of the gravitational field of static spherically symmetric stars in Cartesian coordinates, that is we only consider the (0,0) component of θ_μ^ν. Next, a brief process for the calculation of the energy density of the gravitational field in Cartesian coordinates will be presented.

In order to calculate the energy density of the gravitational field in Cartesian coordinates, first of all, we should get the expression of the metric tensor in Cartesian coordinates. Using the coordinate transformation relation: $x = r\cos\phi\sin\theta$, $y = r\sin\phi\sin\theta$, $z = r\cos\theta$, according to the space-time of a static spherically symmetric star,

$$ds^2 = e^{2\nu}dt^2 - e^{2\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)\,, \tag{7}$$

one can get the metric of a static spherically symmetric star in Cartesian coordinates as follows

$$\begin{aligned} ds^2 \;=\; & e^{2\nu}dt^2 - \Big[e^{2\lambda}\frac{x^2}{r^2} + \frac{x^2z^2}{r^2(x^2+y^2)} + \frac{y^2}{x^2+y^2}\Big]dx^2 \\ & -\Big[e^{2\lambda}\frac{y^2}{r^2} + \frac{y^2z^2}{r^2(x^2+y^2)} + \frac{x^2}{x^2+y^2}\Big]dy^2 - \Big[e^{2\lambda}\frac{z^2}{r^2} + \frac{x^2+y^2}{r^2}\Big]dz^2 \\ & -\Big[e^{2\lambda}\frac{2xy}{r^2} + \frac{2xyz^2}{r^2(x^2+y^2)} - \frac{2xy}{x^2+y^2}\Big]dxdy \\ & -\left(e^{2\lambda}\frac{2xz}{r^2} - \frac{2xz}{r^2}\right)dxdz - \left(e^{2\lambda}\frac{2yz}{r^2} - \frac{2yz}{r^2}\right)dydz\,. \end{aligned} \tag{8}$$

Based on Eq. (8), it is easy to write out the components of the covariant and contravariant metric tensor in Cartesian coordinates. Then according to Eqs. (4) $\sim$ (6), we can obtain [6]

$$\theta_0^0 = (re^{2\nu})^{\frac{1}{2}}(p+\rho)\frac{M(r)}{[r-2M(r)]^{\frac{3}{2}}}\,. \tag{9}$$

where $M(r) = \int_0^r 4\pi r^2\rho dr$ is the mass of a sphere with radius of r. Eq. (9) is just the analytical expression for the energy density of the gravitational field of a static spherically symmetric perfect fluid star in Cartesian coordinates. It is evident that the energy density is positive and rational.

III. NUMERICAL RESULTS AND DISCUSSION

In order to try to understand the magnitude of the energy density of a strong gravitational field, in this part, two kinds of EOS will be employed to calculate the energy density for the gravitational field of neutron stars. In one of the EOS, the neutron stars are mainly composed of neutrons, protons, and electrons; they are called traditional neutron stars (TNS); for the other EOS, the neutron stars are not only composed of the elements of the TNS, but are also composed of hyperons ($\Lambda, \Sigma, \Xi, \Delta$ etc.); they are called hyperon stars (HS). Here the relativistic $\sigma - \omega$ model [9–11] will be adopted to deal with the EOS of the neutron stars. The Lagrangian density of this model is

$$\begin{aligned} L \;=\; & \sum_B \bar{\psi}_B(i\gamma_\mu\partial^\mu + m_B - g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - \frac{1}{2}g_{\rho B}\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu)\psi_B \\ & +\frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - U(\sigma) \\ & -\frac{1}{4}\rho_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu + \sum_l \bar{\psi}_l(i\gamma_\mu\partial^\mu - m_l)\psi_l, \end{aligned} \tag{10}$$

in which $U(\sigma) = a\sigma + \frac{1}{3!}c\sigma^3 + \frac{1}{4!}d\sigma^4$; $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$; $\boldsymbol{\rho}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu$; ψ_B is the field operator of the Baryons B ($B = n$, p for the traditional neutron stars, $B = n$, p, Λ, Σ, Ξ, Δ for the hyperon stars); ψ_l is the field operator of the leptons $l(l = e,\ \mu)$; moreover σ, ω^μ, $\boldsymbol{\rho}^\mu$ are the field operators of the σ-, ω-, ρ-mesons, respectively, and $g_{\sigma B}$, $g_{\omega B}$, $g_{\rho B}$ are the coupling constants between the σ-, ω-, ρ-mesons and the baryons B. In general, the coupling constants between the $\sigma(\omega$ or $\rho)$ meson and the neutron and proton are equal and are decided by the saturated property of nuclear matter ($g_{\sigma n} = g_{\sigma p}$, $g_{\omega n} = g_{\omega p}$) and the symmetry energy of nuclear matter ($g_{\sigma\rho}$, $g_{\omega\rho}$); m_B, m_l, m_i, ($i = \sigma,\ \omega,\ \rho$) are the mass of the baryon, lepton, meson respectively; $\boldsymbol{\tau}$ is the isospin operator. As for leptons, we assume they are a free fermi gas. From this Lagrangian density, the energy-momentum tensor $T^{\mu\nu}$ in the quantum field theory is,

$$T^{\mu\nu} = -Lg^{\mu\nu} + \frac{\partial q_i}{\partial x^\nu}\frac{\partial L}{\partial(\partial q_i/\partial x^\mu)}\,, \tag{11}$$

where q_i is the field of any particle in the Lagrangian density L. As the neutron star matter can be looked on as a perfect fluid, then

$$\langle T^{\mu\nu}\rangle - \langle T^{\mu\nu}\rangle_{vac} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu}, \tag{12}$$

in which $u^\mu = (1, \vec{0})$ in the comoving coordinates. In the 1-loop approximation, the loop's contribution to the propagators of the nucleon and σ meson is considered, and renormalization is used to renormalize the divergent part of the loop contribution. According to Eqs. (10) $\sim$ (12), the energy density ρ and pressure p can be expressed as

$$\rho = \rho_{MF} + \rho_{qf}^B + \rho_{qf}^\sigma\,, \tag{13}$$

$$\begin{aligned} \rho_{MF} \;=\; & \sum_B \frac{\gamma_B}{(2\pi)^3}\int_{K_{FB}} d^3k\tilde{E}_B(k) + \sum_l \frac{\gamma_l}{(2\pi)^3}\int_{K_{Fl}} d^3kE_l(k) \\ & +\frac{1}{2}m_s^2\tilde{\upsilon}^2 - \frac{1}{2}m_\omega^2\tilde{V}_\omega^2 + g_\omega\tilde{V}_\omega\rho + g_\rho\tilde{V}_\rho\rho - \frac{1}{2}m_\rho^2\tilde{V}_\rho^2 \\ & +\lambda_1^{(0)}\tilde{\upsilon} + \frac{1}{3!}\lambda_3^{(0)}\tilde{\upsilon}^3 + \frac{1}{4!}\lambda_4^{(0)}\tilde{\upsilon}^4\,, \end{aligned} \tag{14}$$

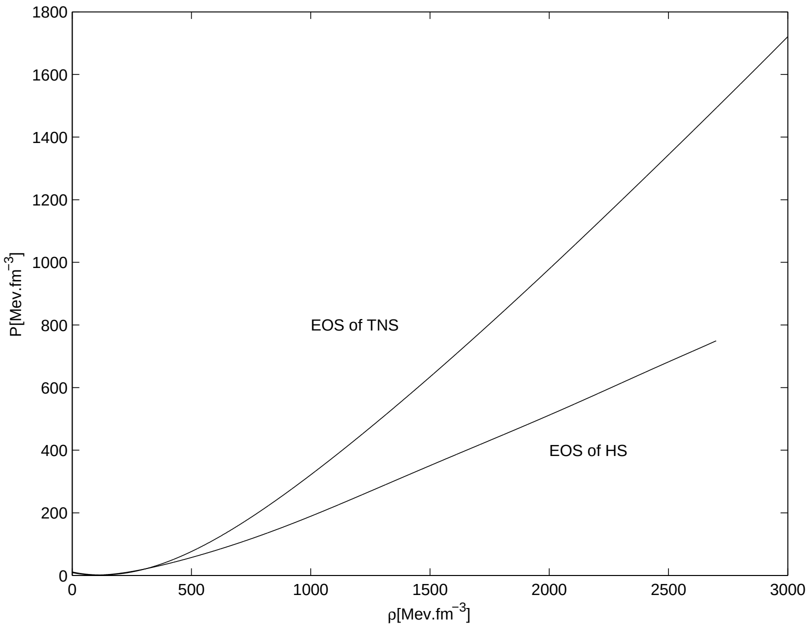

FIG. 1: Equations of state of the traditional neutron stars (TNS) and hyperon stars (HS).

$$\rho_{qf}^{B} = \sum_{B} \frac{\gamma_B}{16\pi^2}\Big[\frac{1}{2}\tilde{m_B}^4 \ln\frac{m_B^2}{\tilde{m}_B^2} + m_B^3(\tilde{m}_B - m_B) + \frac{7}{2}m_B^2(\tilde{m}_B - m_B)^2 + \frac{13}{3}m_B(\tilde{m}_B - m_B)^3 + \frac{25}{12}(\tilde{m}_B - m_B)^4\Big], \tag{15}$$

$$\rho_{qf}^{\sigma} = \frac{m_s^4}{64\pi^2}F\left(\frac{\tilde{m}_s^2}{m_s^2}\right) - \frac{\lambda_3^{(0)^2}}{32\pi^2 m_s^2}\Big[\frac{1}{6}\lambda_3^{(0)}\tilde{v}^3 + \frac{1}{4}(\lambda_4^{(0)} - \frac{1}{72m_s^2}\lambda_3^{(0)^2})\tilde{v}^4\Big], \tag{16}$$

$$F(x^2) = x^4 \ln x^2 - \frac{3}{2}(x^4 - 1) + 2(x^2 - 1), \tag{17}$$

$$P = P_{MF} + P_{qf}^{B} + P_{qf}^{\sigma}, \tag{18}$$

$$P_{MF} = \sum_{B}\frac{1}{3}\frac{\gamma_B}{(2\pi)^3}\int_{k_{FB}} d^3k\frac{k^2}{\tilde{E}_B(k)} + \sum_{l}\frac{1}{3}\frac{\gamma_l}{(2\pi)^3}\int_{k_{Fl}} d^3k\frac{k^2}{E_l(k)} - \frac{1}{2}m_s^2\tilde{v}^2 + \frac{1}{2}m_\omega^2\tilde{V}_\omega^2 + \frac{1}{2}m_\rho^2\tilde{V}_\rho^2 - \lambda_1^{(0)}\tilde{v} - \frac{1}{3!}\lambda_3^{(0)}\tilde{v}^3 - \frac{1}{4!}\lambda_4^{(0)}\tilde{v}^4, \tag{19}$$

$$P_{qf}^{B} = -\rho_{qf}^{B}\,, \tag{20}$$

$$P_{qf}^{\sigma} = -\rho_{qf}^{\sigma}\,, \tag{21}$$

where ρ_{MF} and P_{MF} are the contributions in the mean field approximation, $\rho_{qf}^{B,\sigma}$, $P_{qf}^{B,\sigma}$ are the vacuum fluctuations of the baryon and σ meson in the 1-loop approximation; $\lambda_1^{(0)}$, $\lambda_3^{(0)}$, $\lambda_4^{(0)}$ are the values of λ_1, λ_3, λ_4 in the 1-loop approximation,

$$\lambda_1 = Z_a a + Z_b b\frac{M_n}{g_{\sigma n}} + \frac{1}{2}Z_c c\left(\frac{M_n}{g_{\sigma n}}\right)^2 + \frac{1}{3!}Z_d d\left(\frac{M_n}{g_{\sigma n}}\right)^3\,, \tag{22}$$

$$\lambda_3 = Z_c c + Z_d d\frac{M_n}{g_{\sigma n}}\,, \tag{23}$$

$$\lambda_4 = Z_d d; \tag{24}$$

Z_a, Z_b, Z_c, Z_d are the renormalizing constants; and $\tilde{v}$, $\tilde{V}_\omega$, $\tilde{V}_\rho$ are the expectation values of the σ, ω, ρ meson in the nuclear matter ground state, respectively. In the numerical calculation, we adopt the value of the parameters as follows: $a = -2.1 \times 10^7 MeV^3$, $c = 0.97 \times M_n$, $d = 1277$, $g_s = 6.73$, $g_v = 8.59$, $M_n = 938MeV$, $m_\omega = 783MeV$, $m_\sigma = 550MeV$, $m_\rho = 770MeV$; the incompressibility of nuclear matter is $224MeV$, which is consistent with the experimental result [12]. From Fig. 1, it is clear that the EOS of TNS is stiffer than the EOS of HS.

According to the structure equation of static spherically symmetric stars, which is called the TOV equation [13, 14],

$$\frac{dp}{dr} = -\frac{(p+\rho)[M(r) + 4\pi r^3 p]}{r[r - 2M(r)]}\,. \tag{25}$$

The EOS, the distribution of the neutron star matter, that is, the pressure and the density as a function of radius can be easy obtained, then the masses and radii of neutron stars with different central densities can be calculated; the results are presented in Figs. 2 and 3, respectively. As the EOS of hyperon stars is softer than that of traditional neutron stars, from Fig. 2 one can see that the softer the neutron stars' EOS is, the smaller the mass of the neutron star with the same central density will be. At the neighbouring region when the central densities are larger than 10^{18} kg$\cdot$m^3, the increase of mass will slow down as the central density increases. For a stable neutron star, it must be in the region where the masses increase and the radii decrease. Figs. 2 and 3 show such regions, for HS such a mass region covers a region of 1.35~1.6$M_\odot$, this is just the region of the observational values of the double neutron star binaries, so the double neutron star binaries stand a good chance to be hyperon stars, that is, the matter of these neutron stars abound in hyperons; similarly, for TNS the possible masses cover a region of 1.7~2.3$M_\odot$, this is just the region of the

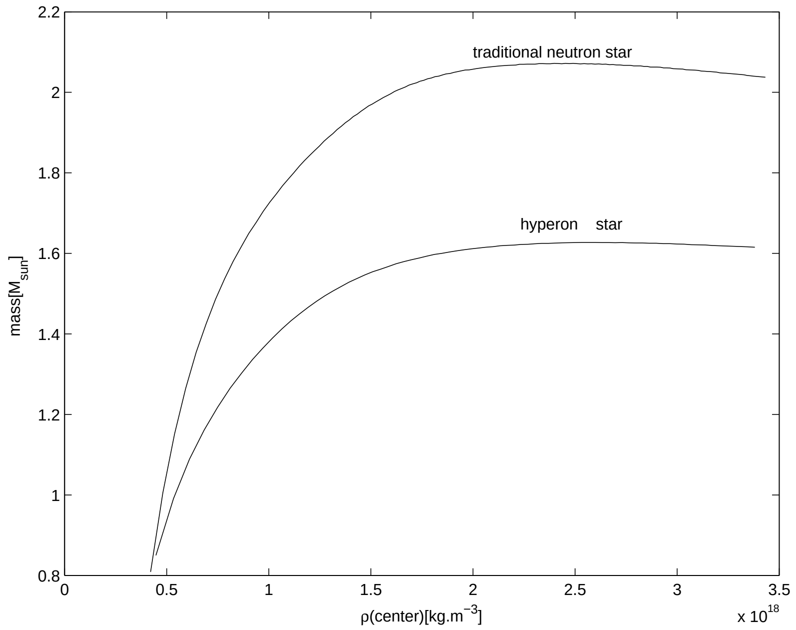

FIG. 2: Masses of TNS and HS as a function of the central density.

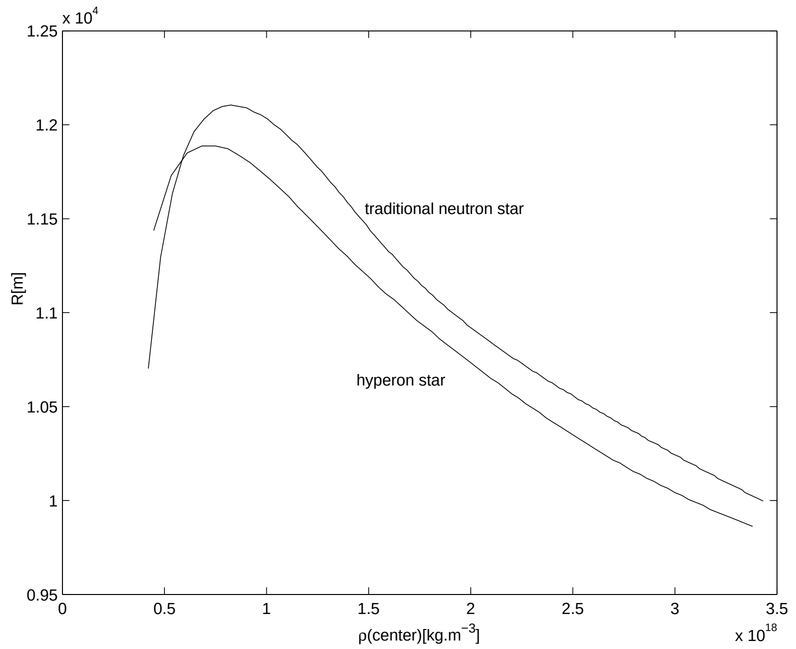

FIG. 3: Radii of TNS and HS as a function of the central density.

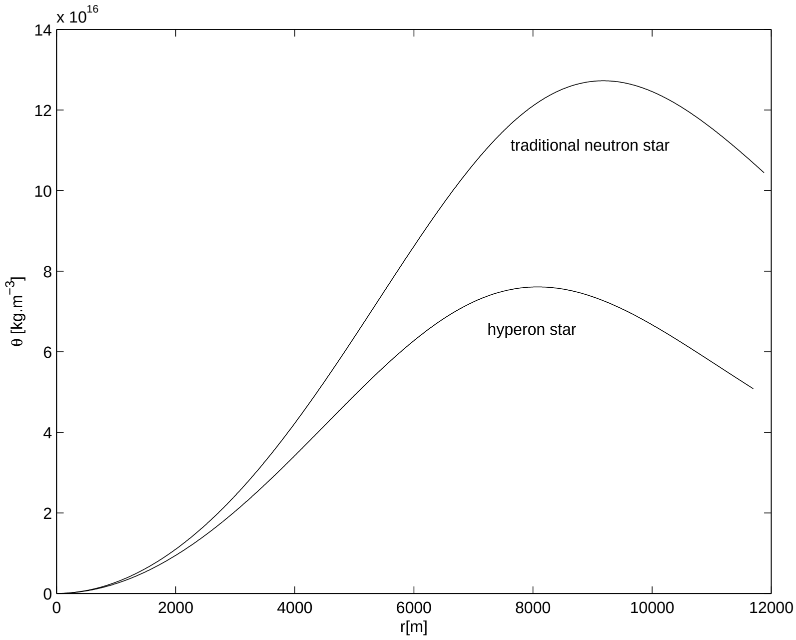

FIG. 4: Energy densities of the gravitational field of HS with a mass of 1.36 $M_{\odot}$ and TNS with a mass of 1.80 $M_{\odot}$, as a function of radius.

observational values of the X-ray binaries, so the neutron stars of X-ray binaries are more like traditional neutron stars, that is, the main matter of these neutron stars are neutrons, protons, and electrons. In our numerical calculation, two observational mass values: 1.36 $M_{\odot}$, which is the typical observational mass value of double neutron star binaries [15], and 1.80 $M_{\odot}$, which is the typical observational mass value of neutrons in X-ray binaries [16, 17], will be adopted for HS and TNS, respectively, to calculate the energy density of the gravitational field.

According to the calculated structure of neutron stars and Eq. (9), the numerical results for the energy density of the gravitational field of neutron stars could be calculated. The results are presented in Figs. 4 $\sim$ 7. Fig. 4 shows the energy densities of the gravitational field of a HS with a mass of 1.36 $M_{\odot}$ and a TNS with a mass of 1.80 $M_{\odot}$ as a function of the star's radius. From this figure, one can see that for both the HS with a mass of 1.36 $M_{\odot}$ and the TNS with a mass of 1.80 $M_{\odot}$, there is a peak of the energy density of the gravitational field in the region with a radius of about 8 km, at the peak, the energy density of the gravitational field of the TNS is 1.213×10^{17} kg·m^3, which is nearly 1.6 times the peak HS value.

Fig. 5 presents the ratio of the energy densities of gravitational fields to the energy densities of matter of HS with a mass of 1.36 $M_{\odot}$ and TNS with a mass of 1.80 $M_{\odot}$ as a function of the star's radius. For TNS, the energy density of the gravitational field can be up to more than 40% of the energy density of the neutron star's matter at the surface.

Figs. 6 and 7 show in three-dimension how the energy density of the gravitational

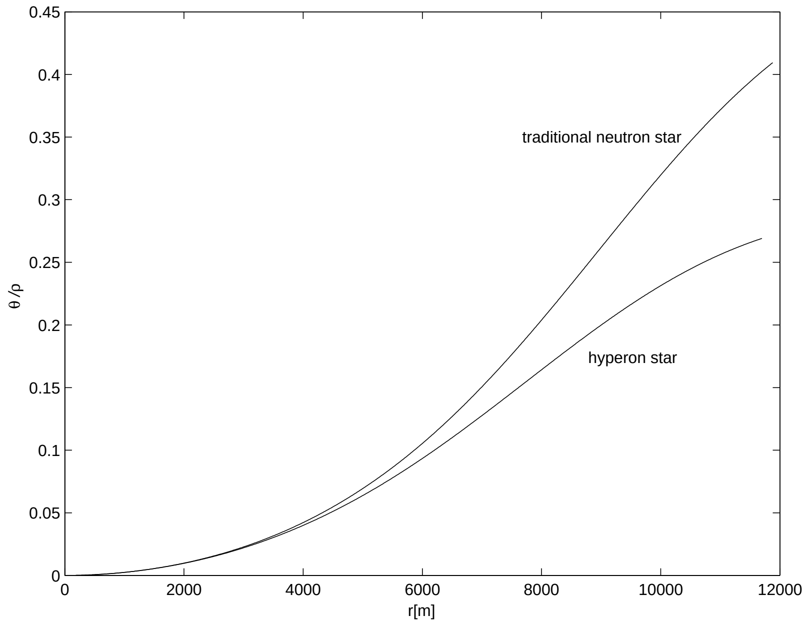

FIG. 5: The ratio of the energy densities of the gravitational field to the energy densities of matters for a HS with a mass of 1.36 $M_{\odot}$ and a TNS with a mass of 1.80 $M_{\odot}$, as a function of radius.

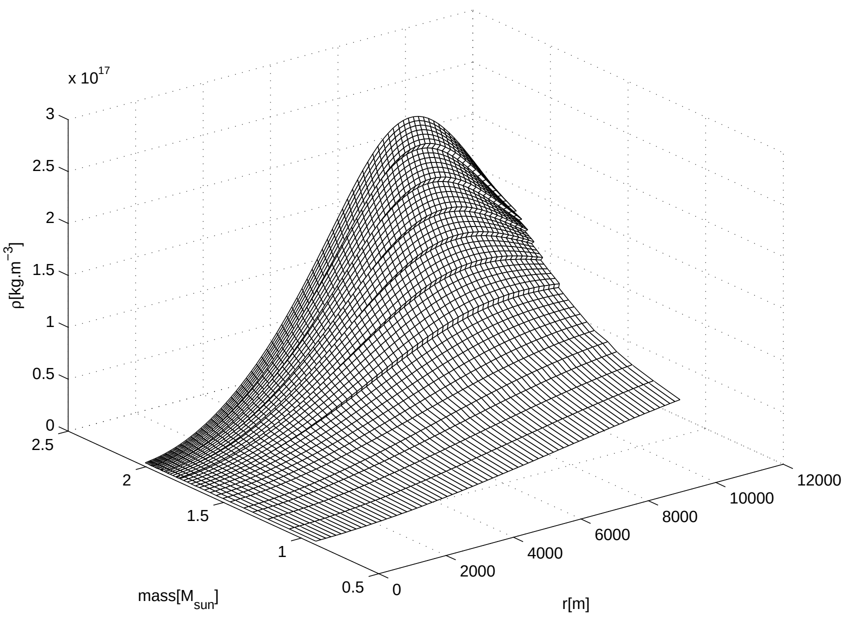

FIG. 6: The energy densities of the gravitational field of a TNS with different total masses as a function of radius.

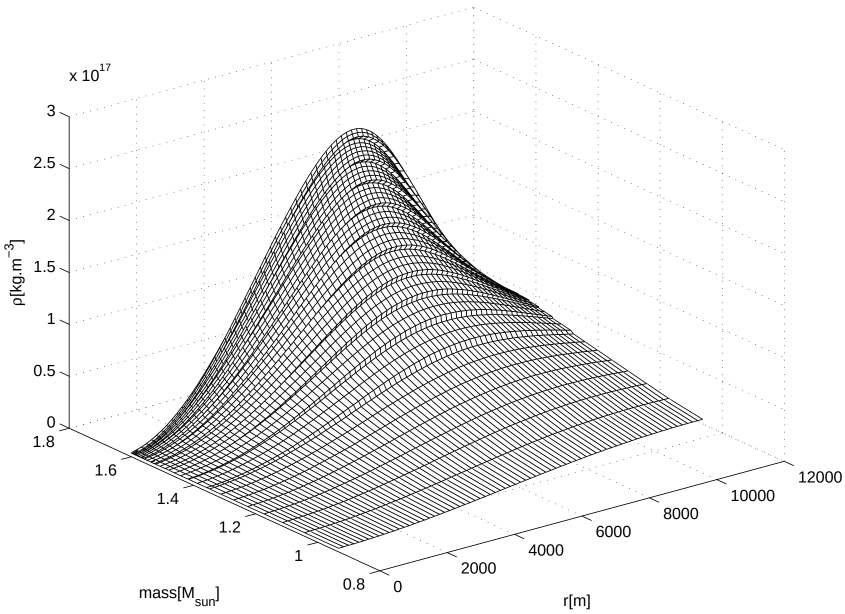

FIG. 7: The energy densities of the gravitational field of HS with different total masses as a function of radius.

field changes follow the radius and the total mass. From these figures, we can see that corresponding to a bigger total mass, there is a peak of the energy density of the gravitational field inside the neutron star, and the peak is around the region with a radius about 8km. For a neutron star with a bigger mass, the energy density of its gravitational field may be near the saturation density of nuclear matter, such a strong gravitational field reminds us that in studying the instability or the gravitational radiation of a neutron star, some important information may be ignored if we deal with them using a weak field approximation.

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