Entanglement Transfer from a Single Atom to Cavity Modes

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We propose a scheme for *entanglement transfer* from an atom to cavity modes. The success rate of our scheme approaches 1 for a broad range of system parameters in the absence of decoherence. Effects of decoherence both from the atom decay and cavity loss on the performance of the scheme are presented and discussed, and a possible strategy to overcome the effects is suggested.

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A large amount of attention has been focused on entanglement creation and manipulation since the discovery that entanglement shared by distant partners is a valuable resource for quantum teleportation [1], cryptography [2], and quantum computation [3]. Among the diverse protocols of entanglement manipulation, entanglement transfer from massive qubits to field modes [4–6] and *vice versa* provides us with a reliable physical interface between fields and qubits. Previous studies in this field were devoted mostly to entanglement transfer from field modes to massive qubits. The reverse process, however, has received relatively little and hence far from exhaustive consideration.

In this paper, we put forward and study the *entanglement transfer* from a single atom to polarized cavity modes; the scheme not only serves as an entangler for the cavity modes, but also acts as an interface between the atom and photons. Suppose that we have a single atom inside a ring cavity with two polarized modes. The atom is maximally entangled in the internal and center-of-mass (external) degrees of freedom. If the atom simultaneously interacts with the cavity modes, then the entanglement shared between the atomic degrees of freedom would be transferred into the modes. From a fundamental point of view, this kind of entanglement transfer activates the entanglement bound in degrees of freedom in a single particle, which has no use in quantum information processing because it is localized. From the side of applications, it provides us with a protocol to entangle two modes in a cavity that may act as a node in distributed networks. Moreover, it converts entanglement from atomic states into quantum states of photons corresponds to a conversion of localized qubits into flying qubits. The latter can be transmitted over long distances and could overcome limitations caused by the spontaneous emission of the atom. More specifically, the photon polarization can be readily measured experimentally, so that an interface between the atom and photons will allow one to measure quantum properties of the atom via the photons.

Now we turn to describe the exact details of our protocol shown schematically in Fig. 1. The system under consideration consists of a V-type atom (Fig. 1(a)) coupling simultaneously to two ring-cavity (Fig. 1(b)) modes σ_+ and σ_-. The σ_- polarized beam

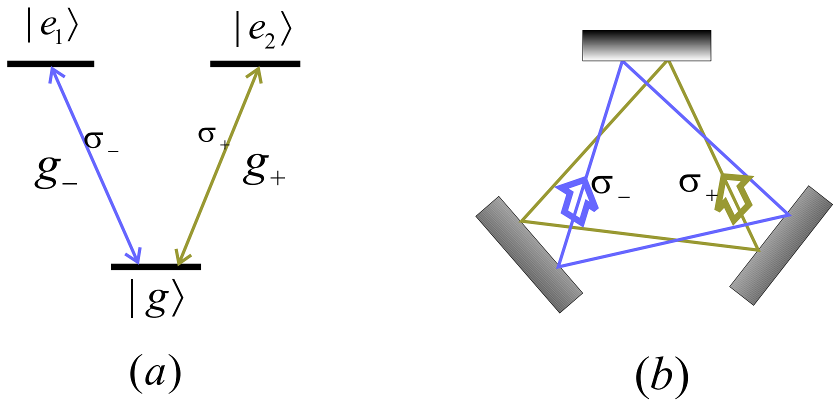

FIG. 1: A V-type atom coupling to cavity modes. The σ_- polarized mode travels along the ring cavity clockwise and couples the ground state $|g\rangle$ to the excited state $|e_1\rangle$, while the counter-propagating beam σ_+ couples the ground state $|g\rangle$ to the excited state $|e_2\rangle$.

travels along the ring cavity clockwise and couples the ground state $|g\rangle$ to the excited state $|e_1\rangle$. The counter-propagating beam is σ_+ polarized and couples the ground state $|g\rangle$ to the excited state $|e_2\rangle$. The Hamiltonian governing such a system reads [7]

$$\begin{aligned} H &= \hbar\omega(|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2|) + \frac{p^2}{2m} + \hbar \sum_{j=+,-} \omega_j a_j^\dagger a_j \\ &+ [\hbar g_- |e_1\rangle\langle g| a_-^\dagger e^{-ikx} + \hbar g_+ |e_2\rangle\langle g| a_+^\dagger e^{ikx} + H.c.]\,, \end{aligned} \tag{1}$$

where k denotes the wavenumber of the two counter-propagating pump beams, p is the momentum operator of the atom which conjugates with the position x of the atom, $a_+^\dagger$ and a_+ ($a_-^\dagger$ and a_-) stand for the creation and annihilation operators of the σ_+ (σ_-) polarized beams, respectively. The first two terms in the Hamiltonian are for the free atom, while the third term is for the cavity; the last terms in Eq. (1) describe the atom-cavity interaction with coupling constants g_- and g_+, respectively. Throughout this paper, we will use the following notations: $|0\rangle$ and $|\pm\hbar k\rangle$ stand for atomic momentum states with momenta 0 and $\pm\hbar k$, respectively; while $|0_\pm\rangle$ and $|1_\pm\rangle$ denote photon states for the $\sigma_\pm$ polarized beams. Let us now discuss a possible realization of entanglement transfer from the atom to the cavity modes. Suppose the atom initially is prepared in the state

$$|\phi_a(0)\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle \otimes |-\hbar k\rangle + |e_2\rangle \otimes |+\hbar k\rangle) \tag{2}$$

with the cavity being in its vacuum state, $|0_{ph}\rangle = |0_+\rangle \otimes |0_-\rangle$. The two counter-propagating beams σ_+ and σ_- would drive the atom into $|g\rangle \otimes |0\rangle$ leaving the cavity in the maximally entangled state

$$|\phi_c\rangle = \frac{1}{\sqrt{2}}(|0_+, 1_-\rangle + |1_+, 0_-\rangle)\,. \tag{3}$$

The state $|\phi_a(0)\rangle$ is maximally entangled for the atom, the entanglement between the internal ($|e_1\rangle$ and $|e_2\rangle$) and external degrees ($|\pm\hbar k\rangle$) of freedom can be prepared by atomic Bragg scattering together with the Ramsey scheme [8]. To see how our scheme works, we transform the Hamiltonian, Eq. (1), by using

$$W = e^{ikx}|e_2\rangle\langle e_2| + e^{-ikx}|e_1\rangle\langle e_1| + |g\rangle\langle g|\,, \tag{4}$$

into a form $H_e = W^\dagger H W$,

$$\begin{aligned} H_e &= \hbar\omega_1|e_1\rangle\langle e_1| + \hbar\omega_2|e_2\rangle\langle e_2| + \frac{p^2}{2m} + \hbar\sum_{j=+,-}\omega_j a_j^\dagger a_j \\ &\quad +[\hbar g_-|e_1\rangle\langle g|a_-^\dagger + \hbar g_+|e_2\rangle\langle g|a_+^\dagger + H.c.]\,, \end{aligned} \tag{5}$$

where $\hbar\omega_{1,2} = \hbar\omega + (\hbar k)^2/(2m) \mp (\hbar pk)/m$ are modified Rabi frequencies involving the Doppler effect $\mp(\hbar pk)/m$ and the photon recoil $(\hbar k)^2/(2m)$. Eq. (5) shows that the associated evolution of the atomic external degree of freedom may be separated from the evolution of the internal degree of freedom. We first consider the ideal case where the atom is in a state with zero momentum and $\omega_+ = \omega_- = \omega_{1,2}$; furthermore we assume that there is at most one photon in the cavity and $g_+ = g_- = g$. With these conditions, in the interaction picture the effective Hamiltonian H_e may be simplified to

$$\mathcal{H} = \hbar g(|G\rangle\langle E| + H.c.)\,, \tag{6}$$

where $|G\rangle$ and $|E\rangle$ are defined as

$$|E\rangle = \frac{1}{\sqrt{2}}(|e_1, 0, 0_+, 0_-\rangle + |e_2, 0, 0_+, 0_-\rangle)\,,$$

and

$$|G\rangle = \frac{1}{\sqrt{2}}(|g, 0, 1_+, 0_-\rangle + |g, 0, 0_+, 1_-\rangle)\,.$$

Clearly, the initially prepared state $|E\rangle$ would evolve to $|G\rangle$ after time $T = \pi/(2g)$. Translated to the original picture, states $|E\rangle$ and $|G\rangle$ correspond to $|E\rangle_r = W|E\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle\otimes|-\hbar k\rangle + |e_2\rangle\otimes|+\hbar k\rangle)\otimes|0_{ph}\rangle$ and $|G\rangle_r = W|G\rangle = |G\rangle$, respectively. $|E\rangle_r$ represents a superposition of states that contain both internal and external degrees of freedom, while $|G\rangle_r$ describes the state of entangled modes. For a system (atom+cavity modes) free of decoherence, the condition $g_+ = g_-$ is not necessary for the scheme, the maximally entangled two-mode states can always be achieved from the initially entangled atomic degrees of freedom with any g_+ and g_-. However $g_+ = g_-$ is the most economic one from the viewpoint of operation time, i.e., minimum coherent dynamics are needed to arrive at the maximally entangled two-mode state. We have performed extensive numerical simulations with the Hamiltonian Eq. (5). When the photon number is restricted to be not more than one, we find the above analytical insights to be completely accurate, i.e., we indeed execute

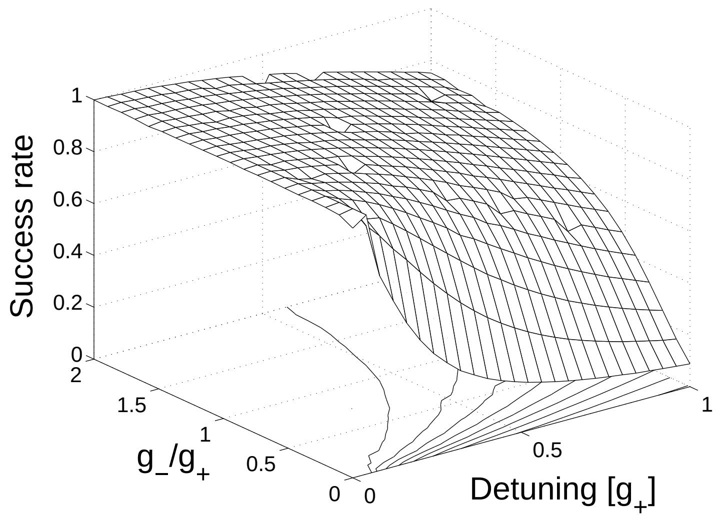

FIG. 2: Success rate *versus* the coupling ratio g_-/g_+ and the detuning defined as $(\hbar\omega_{1,2} - \hbar\omega_\pm)$. The decoherence effects were excluded for this plot.

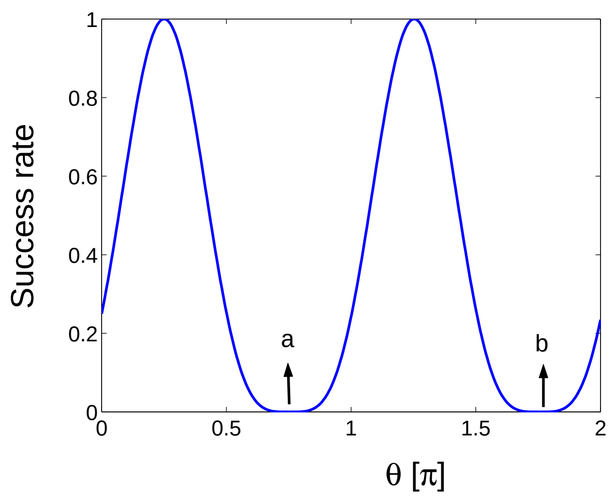

FIG. 3: Success rate *versus* the mixing angle θ with equal coupling constants, i.e., $g_- = g_+$ and zero detuning. a and b indicate the forbidden angles for the entanglement transfer.

the entanglement transfer from the single atom to the cavity modes. In fact, we find that the scheme works with almost perfect performance even beyond the limit that our ideal analysis implies, and the performance is sensitive to the detuning more than to the coupling ratio g_-/g_+.

Fig. 2 shows selected results for the dependence of the entanglement transfer success rate on the system parameters. The success rate is defined as the probability of finding the system in the target state $|G\rangle_r$. Quite promising rates are obtained over a broad range of system parameters. In addition to the system parameters, the success rates also depend on the initial state. An interesting example is that the success rate is zero when the atom takes the initial state

$$|\phi_a(0)\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle \otimes |-\hbar k\rangle - |e_2\rangle \otimes |+\hbar k\rangle)\,.$$

This state decouples from the ground state $|g\rangle|0\rangle$ when $g_+ = g_-$, and thus the entanglement could not be (forbidden) transferred from the atom to the photon states. For a general initial state

$$|\phi_a(0)\rangle = \cos\theta|e_1\rangle \otimes |-\hbar k\rangle + \sin\theta|e_2\rangle \otimes |+\hbar k\rangle\,,$$

the entanglement transfer success rate has been plotted in Fig. 3 as a function of the mixing angle θ, the forbidden angles for entanglement transfer were explicitly indicated in the figure. The other nonideal initial state is an atom having a finite momentum width [9] described by

$$\rho_a(0) = \int_{-\infty}^{\infty} f(p)|\phi_a(p)\rangle\langle\phi_a(p)|dp\,, \tag{7}$$

where $|\phi_a(p)\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle \otimes |p-\hbar k\rangle + |e_2\rangle \otimes |p+\hbar k\rangle)$ and $f(p)$ characterizes the momentum distribution. We assume here that f can be approximated by a Gaussian distribution certered around $p = 0$, $f(p) = 1/(\sqrt{2\pi}\sigma_p)\exp[-p^2/(2\sigma_p^2)]$. The momentum width is given by $\sigma_p^2 = \int p^2 f(p)dp$. It is a measure of the temperature of the atom and affects the success rate through the Doppler effect, the atom with momentum p would experience a detuning of $\Delta = \omega_2 - \omega_+ = \hbar pk/m$ $(\omega_1 - \omega_- = -\hbar pk/m)$ when transitions $|e_2\rangle \longleftrightarrow |g\rangle$ $(|e_1\rangle \longleftrightarrow |g\rangle)$ occur. In preparing the initial state, the width of the momentum distribution can be refined to be less than $\hbar k$, this leads to a very small correction to the performance of our scheme, and hence the scheme is robust against thermal fluctuations.

Now we discuss the effects of decoherence due to both the cavity loss κ and the atomic decay γ. As with any proposal for quantum information processing, ultimately its success depends on being able to complete much coherent dynamics during the decoherence time. In our case, the cavity loss κ and the atomic spontaneous emission γ with limit $|g|^2 \gg \gamma\kappa$ is a challenging pursuit as it is for all quantum computing proposals in cavity QED system [10], nevertheless, it can be expected to be reachable soon with optical cavity QED based technology [11, 12]. In Fig. 4, we display the dependence of the optimal success rate on the cavity decay rate κ and the atomic decay rate γ with $g_+ = g_-$ and $\omega_+ = \omega_- = \omega_{1,2}$. The numerical simulations were performed based on the master equation [13]

$$i\hbar\frac{\partial}{\partial t}\rho \;=\; [H_e, \rho] - i\kappa \sum_{j=+,-} \left(a_j^\dagger a_j \rho + \rho a_j^\dagger a_j - 2a_j \rho a_j^\dagger\right)$$

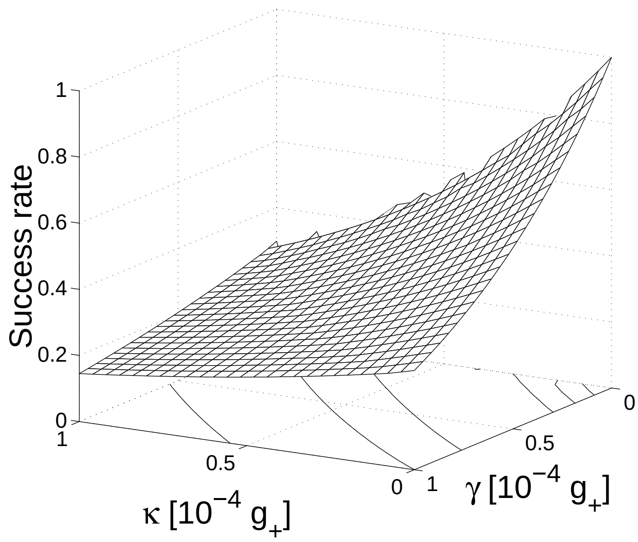

FIG. 4: The decoherence effect. The data were obtained with $g_+ = g_-$ and $\omega_{1,2} = \omega_\pm$.

$$-i\gamma \sum_{m=e_1,e_2} \left(|m\rangle\langle m|\rho + \rho|m\rangle\langle m| - 2|g\rangle\langle m|\rho|m\rangle\langle g|\right). \tag{8}$$

We see that the success rate was greatly lowered due to the cavity loss and atom decay. The problem can be remarkably improved when the limit $|g|^2 \gg \gamma\kappa$ is lifted. In fact our model maps perfectly into an ion trap setup [14], where prior entanglement shared between the internal and external degrees of freedom would be $\frac{1}{\sqrt{2}}(|e_1\rangle|n-1\rangle + |e_2\rangle|n+1\rangle)$ instead of $\frac{1}{\sqrt{2}}(|e_1\rangle|-\hbar k\rangle + |e_2\rangle|+\hbar k\rangle)$ for the atom inside the cavity. The two polarized laser beams would drive the ion to undergo red and blue sideband Raman transitions to the ground state $|g\rangle|n\rangle$. As in all proposals where photons play the role of information carrier, the performance of our scheme is ultimately determined by the lifetime of the photons. This alternative proposal in the ion trap setup promises to solve this problem, and the influence of the atom decay may be suppressed by employing adiabatic passage and appropriately designed atom-field couplings. This scheme was depicted in Fig. 5, where $|g_1\rangle$, $|g\rangle$ and $|g_2\rangle$ are long-lived atomic states. Our proposal starts with preparing an initially entangled state $1/\sqrt{2}(|g_1\rangle|-\hbar k\rangle + |g_2\rangle|+\hbar k\rangle$. The adiabatic passage with properly designed pulses Ω_1 and Ω_2 would allow one to transfer the entanglement share among the atomic degrees of freedom to the field modes.

In summary, we have studied the transfer of entanglement from atomic degrees of freedom to photon polarizations. Although the scheme is not good for entanglement preparation, it serves as a possible realization for entanglement transfer. We have shown the dependence of the success rate as a function of the detuning and the coupling ratio and have given the condition under which this entanglement transfer success rate is maximal. This offers the possibility to effectively test Bell's inequalities and measure the entangle-

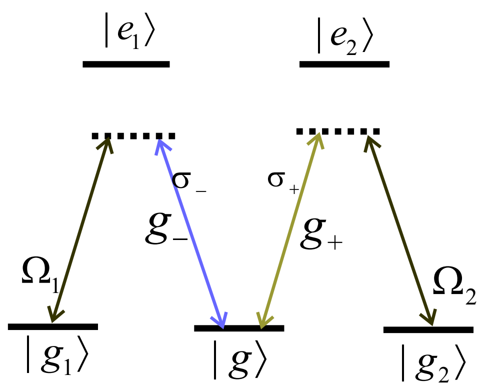

FIG. 5: A five-level atom interacting with cavity modes and laser pulses.

ment for the atoms. We have also discuss the influence of decoherence, and the dependence of the success rate on the cavity loss and atom decay has been presented and discussed. Finally, the scheme we have proposed here can also be realized in the ion trap setup, where the prior entanglement was shared for the atom in different internal levels associated with sidebands.

Acknowledgments

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References

[1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[2] M. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, Rev. Mod. Phys. **74**, 145 (2003).
[3] M. A. Nielsen, and I. L. Chuang, Quantum computation and quantum information (Cambridge University press, Cambridge, 2000).
[4] Y. Omar, N. Paunković, S. Bose, and V. Vedral, Phys. Rev. A **65**, 062305 (2002).
[5] D. Petrosyan, G. Kurizki, and M. Shapiro, Phys. Rev. A **67**, 012318 (2003).
[6] M. Perternostro, W. Son, M. S. Kim, G. Falci, and G. M. Palma, Phys. Rev. A **70**, 022320 (2004).
[7] M. O. Scully and M. S. Zubairy, Quantum optics (Cambridge University press, 1997). Taking the atomic motion into account, the Hamiltonian which describes the interaction of a radiation field $\boldsymbol{E}$ with a single-electron atom may be written as (in the dipole approximation), $H = H_A + H_F - e\boldsymbol{r_e} \cdot \boldsymbol{E}$, where H_A and H_F are the energies of the atom and the radiation field, respectively. $\vec{r_e}$ is the position vector of the electron. The energy of atom takes the form, $H_A = P^2/2m + V(\boldsymbol{R} - \boldsymbol{r_e}) + p_e^2/2m_e$. Here P and p_e stand for the momentum operators for

the atom (exactly speaking, it is for the nucleus) and electron, respectively. $\boldsymbol{R}$ is the position operator of the atom. By the Born-Oppenheimer approximation, the energy of the atom reduces to $H_A = P^2/2m + \sum_i E_i|i\rangle\langle i|$, where $|i\rangle$ and E_i are eigenstates and corresponding eigenvalues of $[V(\boldsymbol{R} - \boldsymbol{r_e}) + p_e^2/2m_e]$, respectively. In the dipole approximation, the field $\boldsymbol{E}$ is assumed to be uniform over the whole atom, however, it depends on the position of the atom. This finding together with the Rotating-Wave-Approximation (RWA) yields the Hamiltonian Eq. (1).

[8] S. Kunze, S. Dürr, and G. Rempe, Europhys. Lett. **34**, 343 (1996); S. Dürr, T. Nonn, and G. Rempe, Nature **395**, 33 (1998).

[9] A. Aspect, E. Arimondo, R. Kaiser, N. Nansteenkiste, and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **6**, 2112 (1989).

[10] P. Bertet, S. Osnaghi, P. Milman, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **88**, 143601 (2002).

[11] C. J. Hood, T. W. Lynn, A. C. Doherty, A. S. Parkins, and H. J. Kimble, Science **287**, 1447 (2000); A. C. Doherty, T. W. Lynn, C. J. Hood, and H. J. Kimble, Phys. Rev. A **63**, 013401 (2001).

[12] P. W. H. Pinkse, T. Fischer, P. Maunz, and G. Rempe, Nature **404**, 365 (2000); M. Hennrich , T. Legero, A. Kuhn, and G. Rempe, Phys. Rev. Lett. **85**, 4872 (2000).

[13] C. W. Gardiner and P. Zoller, Quantum Noise (Springer, 2000).

[14] S. Schneider, D. F. V. James, and G. J. Milburn, J. Mod. Opt. **47**, 499 (2000).