

## Some Unfinished Thoughts on Strong Yukawa Couplings

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Yukawa couplings of electroweak Goldstone bosons can be inferred from experiment, but the existence of an elementary Higgs boson is not yet an established fact. If a sequential chiral quark generation does exist, it would bring us now into the strong Yukawa coupling regime. Guided by a Bethe–Salpeter equation approach, we postulate that the leading collapse state, the (heavy) isotriplet and color-singlet  $\pi_1$  meson, becomes the Goldstone boson  $G$  itself. Viewing it as a deeply bound state, a gap equation is constructed. This “bootstrap” picture for electroweak symmetry breaking relies on strong Yukawa coupling, without providing any theory of the latter.

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### I. BREACHING UNITARITY BOUNDS

Particle physics has seen a great leap forward in 2011: we celebrate the enormous success of the LHC, running at 7 TeV. But apprehension arose: No New Physics was seen, while enormous parameter space was excluded for the Higgs boson, the holy grail of LHC physics. To quote a sage [1]: “Our field seems to be approaching a definite point of reckoning. But will it lead us to enlightenment, or to disillusionment and chaos?”

In a dramatic way, the strong hint [2] as of late July for the (dreaded by most)  $\sim 140$  GeV Higgs boson diminished [3] by the end of August, and dropped from view [4] by December 13. In its stead, there is a mild hint at 125 GeV, where, though chastised by the (forced-by-LHC) retreat to above TeV scale, the supersymmetry camp draws momentary warmth.

In this note we shun the 125 GeV Higgs possibility (we would know by end of 2012 whether it shares the fate of the 144 GeV hint of July 2011), but consider the other option:  $m_H > 600$  GeV [4]. In fact, we would shun the whole idea of the Higgs boson as an elementary particle, and *return to the basics of experiment-based knowledge*. Simply put, we do not have any firm experimental knowledge that the Higgs boson even exists. Rather, it is the most economical theoretical construction for electroweak symmetry breaking (EWSB), at the cost of bringing in many theoretical problems, such as quadratic divergences and the resulting hierarchy problem.

The physical bound of  $m_H > 600$  GeV would imply one is close to the onset of strong  $W_L W_L$  scattering [5]. The approach to another “unitarity bound” (UB) is also imminent: strong  $Q\bar{Q}$  (and  $QQ$ ) scattering at high energies for heavy chiral quark  $Q$  [6]. The CMS experiment has searched for the 4th generation  $t'$  and  $b'$  quarks, and the stringent bounds [7], at  $\sim 500$  GeV, are not far from the UB of  $\sim 500$ – $600$  GeV for  $Q\bar{Q}$  scattering.

The 4th generation faces the difficulty of an unseen Higgs [1, 8], given that it tends to

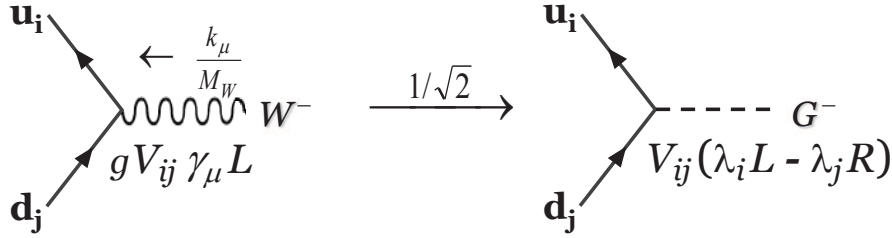


FIG. 1: Deriving the Yukawa coupling, *i.e.*, Goldstone boson coupling to quarks, from purely left-handed gauge couplings.

enhance Higgs production via gluon fusion by an order of magnitude. But taking  $m_H > 600$  GeV neutralizes this stigma [9]. Instead, could the near UBV (UB violation) of strong  $W_L W_L$  scattering and strong  $Q\bar{Q}$  (and  $QQ$ ) scattering be correlated? Could the *strong Yukawa couplings* of a new sequential heavy chiral quark generate [10] EWSB itself? This is the theme we shall explore.

## II. FROM GAUGE TO YUKAWA COUPLINGS

Given the curious absence of evidence for a Higgs boson, let us recall the firm facts from experiment.

First, we know [11] that  $q/l$  are pointlike to  $10^{-18}$  m, and are governed by the  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  gauge dynamics. Chromodynamics would not be our concern, but it is important to emphasize that, unlike the 1970s and early 1980s, the  $SU(2)_L \otimes U(1)_Y$  *chiral* gauge dynamics is now experimentally established. We know that quarks and leptons come in left-handed weak doublets and right-handed singlets, and for each given electric charge, they carry different hypercharge  $Y$ .

Second, the weak bosons are found [11] to be massive,  $M_W = \frac{1}{2}gv$ , where  $g$  is the measured  $SU(2)_L$  weak coupling, and  $v^2 = 1/\sqrt{2}G_F$  the vacuum expectation value. Hence, spontaneous breaking of  $SU(2)_L \otimes U(1)_Y$  symmetry (SSB) is also experimentally established.

Third, all fermions are observed [11] to be massive. These masses also indicate EWSB, since they link left- and right-handed fermions of same electric charge, but different  $SU(2)_L$  and  $U(1)_Y$  charges. We shall not invoke the elementary Higgs boson for mass generation, as it is not yet observed experimentally.

At this point we need to acknowledge the important theoretical achievement of renormalizability [12] of non-Abelian gauge theories, which allowed theory-experiment correspondence down to per mille level precision. Worthy of note is that the proof of renormalizability is based on the Ward identities and [13] is unaffected by SSB, *i.e.*, the underlying symmetry properties are not affected. From this, we now argue [14] for the existence of Yukawa couplings as an experimental fact.

With proof of renormalizability, we choose the physical unitary gauge, hence there are no would-be Goldstone bosons (or unphysical scalars), only massive gauge bosons [15]. The  $\frac{k_\mu k_\nu}{M_W^2}$  part of a  $W$  boson propagator reflects longitudinal  $W$  boson propagation (which is the would-be Goldstone bosons that got “eaten”). If we take a  $\frac{k_\mu}{M_W}$  factor and contract with a  $d_j \rightarrow u_i$  charged current, as illustrated in Fig. 1, simple manipulations give (dropping  $V_{ij}$  for convenience),

$$\begin{aligned} \frac{g}{\sqrt{2}} \frac{k_\mu}{M_W} L &= \frac{g}{\sqrt{2}} \frac{\not{p}_i - \not{p}_j}{M_W} L = \frac{g}{\sqrt{2}} \frac{m_i L - m_j R}{M_W} \\ &= \sqrt{2} \left( \frac{m_i}{v} L - \frac{m_j}{v} R \right) \\ &\equiv \lambda_i L - \lambda_j R, \end{aligned} \tag{1}$$

where we have inferred

$$\lambda_Q \equiv \frac{\sqrt{2} m_Q}{v}, \tag{2}$$

as the effective  $W_L$ , or Goldstone boson  $G$  coupling to quarks, which is nothing but the familiar Yukawa coupling. We have used the equation of motion in the second step of Eq. (1), but this is justified since we work in the broken phase of the real world, and we know that all quarks are massive.

From Fig. 1 and Eqs. (1) and (2), we see that from the experimentally established left-handed gauge coupling, the Goldstone boson couples via the usual Yukawa coupling. The Goldstone bosons of EWSB pair with the transverse gauge boson modes to constitute a massive gauge boson, the Meissner effect, but the important point is that we have not introduced a physical Higgs boson in any step. Unlike the Higgs boson, SSB of electroweak symmetry is an experimentally established fact. The Goldstone bosons couple with Yukawa couplings proportional to fermion mass.

We have kept a factor  $V_{ij}$  in Fig. 1. Recall that the Kobayashi–Maskawa (KM) formalism [16] for quark mixing deals with massive quarks, or equivalently the existence of Yukawa matrices, and the argument remains exactly the same. A vast amount of flavor and  $CP$  violation (CPV) data overwhelmingly supports [11] the 3 generation KM picture. For example, the unique CPV phase with 3 quark generations can so far explain all observed CPV phenomena. These facts further attest to the existence of Yukawa couplings from their dynamical effects, but again do not provide any evidence for the existence of the Higgs boson.

### III. YUKAWA BOUND STATES: A POSTULATE

Based on experimental facts and the renormalizability of electroweak theory, we have “derived” Yukawa couplings from purely left-handed gauge couplings in the previous section without invoking an explicit Higgs sector, at least not at the empirical, heuristic level. We

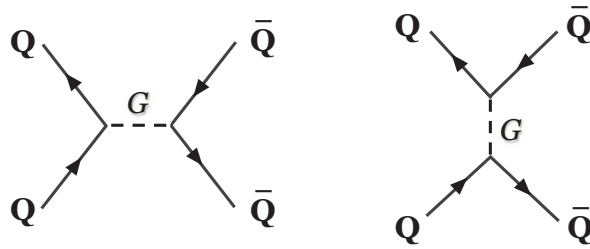


FIG. 2: Scattering diagrams for  $t$ - and  $s$ -channel Goldstone boson  $G$  exchange between  $Q$  and  $\bar{Q}$  quarks. Analogous diagrams can be drawn for  $g$  (and  $H$ ) exchange.

turn now to a more hypothetical situation: Could there be more chiral generations? Since we already have three, the possibility that there exists a fourth generation of quarks should not be dropped in a cavalier way. Indeed, there has been some resurgent interest [7, 17] recently, and, as argued in the introductory section, one should press on when considering the “heavy Higgs” scenario. What we do know is that the 4th generation  $t'$  and  $b'$  quarks should be suitably degenerate to satisfy electroweak constraints on the  $S$  and  $T$  variables [18]. A “heavy isospin” is in accord with the custodial  $SU(2)$  symmetry.

With  $m_{t'} \cong m_{b'} \equiv m_Q \gtrsim 500$  GeV [7], their Yukawa couplings are already 3 times stronger than the top quark, hence stronger than all gauge couplings. There has been two complementary studies of strong Yukawa bound states. The first approach is along traditional lines of relativistic expansion [19]. Ignoring all gauge couplings except QCD, and taking the heavy isospin limit (hence  $Q$  represents a 4th generation quark doublet, and  $G$  a triplet of Goldstone bosons), the  $t$ - and  $s$ -channel Goldstone exchange diagrams are depicted in Fig. 2, with corresponding diagrams for  $g$  as well as  $H$  exchange (Ref. [19] did not put in  $s$ -channel gluon exchange).

The heavy  $\bar{Q}Q$  mesons form isosinglets and isotriplets, and can be color singlet or octet. We borrow the notation from hadrons and call these states  $\eta_1, \omega_1, \pi_1, \rho_1$  and  $\eta_8, \omega_8, \pi_8, \rho_8$ , respectively. Ref. [19] used a variational approach, with radius  $a_0$  as parameter. It was found that, for color singlet  $\omega_1$  ( $\rho_1$ ),  $a_0/a_{\text{QCD}} \sim 1$  for  $m_Q$  below 400 (540) GeV, but above which  $a_0$  suddenly precipitates towards tiny values. For  $\eta_1$  ( $\pi_1$ ) the radius mildly decreases (increases) from 1, with a reverse trend for the binding energy, hence it remains QCD-bound.

To understand this, note that the  $t$ -channel Goldstone exchange for  $\eta_1$  is repulsive, while the  $s$ -channel Goldstone exchange, which contributes only to  $\pi_1$ , is also repulsive. However, the sudden drop in  $\omega_1$  and  $\rho_1$  radii is due to the trial wave function suddenly sensing a lower energy at tiny radius due to  $t$ -channel Goldstone exchange: the strong Yukawa coupling has wrested control of binding from the Coulombic QCD potential. The QCD binding energy is only a couple of GeV, but the sudden drop in radius leads to a sharp rise in binding energy, giving rise to a kink. The relativistic  $v/c$  expansion fails just when it starts to get interesting. For color octet states, QCD is repulsive, so  $\eta_8$  does not bind. In Ref. [19], the  $\omega_8$  and  $\pi_8$  states are degenerate, with sudden shrinking of radius

occurring around 530 GeV, but the  $s$ -channel QCD effect, left out in Ref. [19], should push the  $\omega_8$  upwards; the  $\rho_8$  state does not shrink until later.

Given that a relativistic expansion breaks down, a truly relativistic approach is needed. Such a study, based on a Bethe–Salpeter (BS) equation [20], was pursued around the time of demise of the SSC. The BS equation is a ladder sum of  $t$ - and  $s$ -channel diagrams of Fig. 2, where the  $\bar{Q}Q$  pair forms a heavy meson bound state. While the ladder sum of  $t$ -channel diagrams are intuitive, a problem emerges for the  $s$ -channel, which contributes only to  $\pi_1$ ,  $\omega_8$ , and  $\sigma_1$  (same quantum numbers as  $G$ ,  $g$ , and  $H$ , respectively). Rather than a triangle loop, the  $s$ -channel loop appears like a self-energy hence potentially divergent, while the momentum carried by the exchanged boson is the bound state mass itself. One could not formally turn the integral equation into an eigenvalue problem. This was resolved in Ref. [20] by a subtraction at fixed external momentum, which in effect eliminates all  $s$ -channel diagrams. Ref. [20] then solved the BS equation numerically using several different approximations, which, in addition to the approximate nature of the BS equation itself, illustrates the uncertainties. Still, unlike Ref. [19], the bound state masses drop smoothly below  $2m_Q$  as  $m_Q$  increases, showing no kink, which is an improvement. However, a generic feature is *collapse*: bound state masses tend to drop sharply to zero at some high  $m_Q$ , and would naively turn tachyonic.

Before we elaborate further about collapse, as well as issues regarding subtracting out  $s$ -channel contributions, we mention that a relative conservative study [21] of strong Yukawa bound state phenomenology was conducted using the BS equation approach as a guide. The mass range selected for study was  $m_Q \in (500, 700)$  GeV, where one would already have strong binding energy of order 100 GeV, but still safely away from the region of collapse, hence one could gainfully use the numerics of Ref. [20]. Without solving the bound state problem in numerical detail, the meson decay constant and other parameters were employed to discuss LHC phenomenology in the near future.

Here, we do not pursue the phenomenology, but wish to address more fundamental issues. Although the *de facto*  $s$ -channel subtraction made by Ref. [20] appeared reasonable on formal grounds, the contrast with the relativistic expansion is striking: the Goldstone  $G$  exchange in the  $s$ -channel leads to a specific repulsion [19] for  $\pi_1$  heavy mesons, disallowing it to shrink suddenly like the otherwise analogous  $\omega_1$ . But after subtracting the  $s$ -channel, Ref. [20] finds the  $\pi_1$  as the *most attractive channel* (MAC), more so than  $\omega_1$ . Together with the tendency towards collapse for large enough  $m_Q$  (equivalently  $\lambda_Q$ ), this means that the  $\pi_1$  meson would be the first to drop to zero and turn tachyonic. That this occurs for the channel that experiences repulsion when  $2m_Q$  is far lower than collapse values (à la  $\omega_1$  which has no  $s$ -channel effect) seems paradoxical. Does this falsify the whole approach, or else what light does this shed? And how is it related to  $s$ -channel subtraction?

With experimental bounds [7] for 4th generation quarks entering the region of deep(er) binding, we offer a self-consistent view that may seem a bit radical. Clearly, around and below 500 GeV mass, or  $2m_Q \lesssim 1$  TeV, there could still be some repulsion due to  $s$ -channel  $G$  exchange. But since we did not introduce any elementary Higgs doublet, the Goldstone boson  $G$  should perhaps be viewed as a  $\bar{Q}Q$  bound state. Hence, we

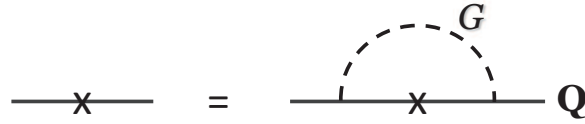


FIG. 3: Gap equation for generating a heavy quark mass from a Goldstone boson loop.

Postulate :  $\pi_1 \equiv G$ , i.e. collapse is a precursor to dynamical EWSB, and the first mode to collapse becomes the Goldstone mode.

Although the full validity of the BS equation may be questioned, it is known [22] that “the appearance of a tachyonic bound state leads to instability of the vacuum”, which is “resolved by condensation into the tachyonic mode”. Our Postulate removes the equation for  $\pi_1$  self-consistently, and provides some understanding of the  $s$ -channel subtraction: a  $\pi_1/G$  boson carrying  $p^2 \sim (2m_Q)^2$  would no longer be a bound Goldstone boson in the  $s$ -channel. Without an elementary Higgs boson, there is no  $\sigma_1$  channel subtraction, while for heavy enough  $m_Q$  (so  $\pi_1$  has turned Goldstone) one can treat QCD effects as a correction, *after* solving the  $\omega_8$  bound state problem, without need of subtracting  $s$ -channel  $g$  exchange. The self-consistent MAC behavior of the  $\pi_1$  channel seems like a reasonable outcome of the Goldstone dynamics, as implied by the gauge dynamics.

It may now appear that EWSB is some kind of a “bootstrap” from “massive chiral quarks” with large Yukawa coupling as seen in broken phase.

#### IV. A GAP EQUATION WITHOUT HIGGS

Motivated by the previous heuristic discussion, we construct a gap equation for the dynamical generation of heavy quark mass without invoking the Higgs boson.

Connecting two of the  $Q$  or  $\bar{Q}$  lines in Fig. 2, one gets the self-energy for  $Q$  by  $G$  exchange, and readily arrives at the gap equation as depicted in Fig. 3. One treats both  $Q$  and  $G$  as massless at the diagrammatic level. If the quark mass  $m_Q$ , represented by the cross  $X$ , could be nonzero, then one has dynamical chiral symmetry breaking, which is equivalent to EWSB!

Such a gap equation was constructed recently from a different, and in our view more *ad hoc*, theoretical argument. In Ref. [23], an elementary Higgs doublet is assumed together with a 4th generation. Motivated by their earlier study [24], where some UV fixed point (UVFP) behavior was conjectured, these authors pursued dynamical EWSB via a Schwinger-Dyson equation that is rather similar to our Fig. 3. However, perhaps in anticipation of the UVFP that might develop at high energy [24], they put in by hand a *massless* Higgs doublet, hence a scale invariant theory to boot. It is the *massless* Higgs doublet that runs in the loop, replacing our Goldstone boson  $G$ . The massless nature of the Higgs doublet appears *ad hoc*, and the paper defers the discussion of the physical Higgs spectrum

for a future work.

In contrast, our Goldstone boson  $G$ , identified as the collapsed  $\pi_1$  state as it turns tachyonic, is strictly massless in the broken phase. In the gap equation of Fig. 3, we speculate that the loop momentum should be cut off around  $2m_Q$ , rather than some “cut-off” scale  $\Lambda > 2m_Q$ . In so doing, we bypass all issues of triviality that arise from having  $\Lambda$  approaching  $2m_Q$ . What happens at scales above  $2m_Q$  is to be studied by experiment.

Here we remark that the first, elementary Higgs of Ref. [23],  $\pi$  and  $\sigma$ , are our bound state Goldstone bosons, and indeed we should have a  $\sigma$ -like massive broad bound state that could mimic the heavy Higgs boson. Their second Higgs doublet, in the form of  $t'$  and  $b'$  bound states, would be excitations above the  $\pi_1$  and  $\sigma_1$  for us, likely rather broad. We think that their claimed third doublet, that of bound  $\tau'$  and  $\nu'_\tau$ , may not be bound at all, as their Yukawa couplings may not be large enough.

The gap equation illustrated in Fig. 3 actually links to a vast literature on strongly coupled, scale-invariant QED. It is known that such a theory could have spontaneous chiral symmetry breaking when couplings are strong enough. Detailed numerical study would be deferred to a future report, but Ref. [23] did provide some numerical study of their gap equation, which, up to some numerical factor, should be similar to ours. So let us offer some remarks before concluding.

The integral gap equation can be cast into differential form [25, 26] plus boundary conditions. A critical value of  $\lambda_Q \sim \sqrt{2}\pi$  seems to be required, which links to  $m_Q \sim 770$  GeV. This is above the current LHC bounds [7], but not so far away! The  $\bar{Q}Q$  condensate, the vacuum expectation value  $v$  (the “pion decay constant”) can all be in principle computed [27]. The 770 GeV value is likely a lower bound on  $m_Q$  for EWSB. Furthermore, lacking the attraction of the scalar Higgs boson as compared to Ref. [23], we suspect that the effective Yukawa coupling, hence critical  $m_Q$ , is likely larger.

Although our line of thought may seem constructed, we have developed a self-consistent picture where EWSB from large Yukawa coupling may be realized with some confidence—all without assuming an elementary Higgs boson. We have not yet really touched on the  $\sigma_1$  meson, which would be the heavy Higgs boson. However, our scenario is to have Goldstone bosons as strongly (and tightly) bound “Cooper pairs” of very heavy quarks, which may please Nambu.

## V. DISCUSSION AND CONCLUSION

We have adhered closely to an empirical approach and collected several observations on the electroweak sector and the possible mechanism of its breaking. We showed that Yukawa couplings exist, as the couplings of electroweak Goldstone bosons, or longitudinal vector bosons, to fermions, which were deduced from purely left-handed gauge couplings, utilizing the fact that both the vector gauge bosons and the charged fermions are all massive. This was done without any reference to the existence of an elementary Higgs doublet. With the backdrop that both the physical “Higgs” particle and the possible existence of an extra quark generation seem to involve strong couplings, we discussed Yukawa-bound  $\bar{Q}Q$

mesons. Taking a cue from the possible collapse of such states at *large* Yukawa coupling, we postulate that the collapsed MAC state, the  $\pi_1$  isotriplet, color-singlet meson is the Goldstone boson  $G$  (or  $W_L$ ) itself. This  $\pi_1$  Goldstone boson is a rather tightly bound (very small radius) state. From this picture, we constructed a simple gap equation that is quite similar to strongly coupled, scale-invariant QED, which is known to exhibit dynamical chiral symmetry breaking at large coupling. A rough estimate of the minimal critical  $m_Q \sim 800$  GeV was suggested.

In essence, we suggest a “bootstrap” picture where both the Goldstone boson, and the heavy quark mass, are generated by a strongly coupled gap equation. The discussion has been heuristic, and the dynamical EWSB is rooted in the existence of a strong Yukawa coupling, without offering any theory for this coupling, except that the existence of Yukawa couplings is based on experiment. Unlike the Nambu–Jona-Lasinio (NJL) model [28], there is no assumption of an effective 4-quark operator. Instead, one utilizes nothing but the Yukawa coupling itself. In the NJL model, the Goldstone boson is a ladder sum of quark-antiquarks interacting through the effective 4-quark operator, but in our case, the interaction is through the Goldstone boson itself, hence closer to a bootstrap model. It would be interesting to explore further the similarities and interconnections of our picture with the NJL model approach [29].

Chiral symmetry breaking in hadron physics is realized by QCD. There are striking differences between QCD (hence technicolor-like models of EWSB) and strong Yukawa induced dynamical EWSB: the heavy quark  $Q$  is not confined. Also, in the QCD picture, the physical pion is still a stringy state. For our  $\pi_1$ , we do not know how it would be finally realized in an ultimate theory (that would explain the origin of Yukawa couplings).

There is an experimental perspective on our gap equation. The longitudinal component of the electroweak vector boson,  $V_L$ , is the Goldstone boson  $G$ . The striking success of the Standard Model against all other alternative, New Physics theories, after collecting two times  $5 \text{ fb}^{-1}$  data at the 7 TeV LHC, makes clear that there may be no other object around or not too far above the weak scale. Thus, in the gap equation of Fig. 3, the Goldstone boson loop indeed seems to sum up all dominant effects, if there exist some yet unseen very heavy quark  $Q$ . Thus, this view of dynamical EWSB is consistent with the 2011 LHC data, and might be testable in the not so distant future.

In conclusion, with heuristic arguments of the physical nature of Yukawa couplings, but without touching on an elementary Higgs sector, by introducing large Yukawa couplings through an extra generation of chiral quarks, we have illustrated how the electroweak symmetry may be dynamically broken, where the gap equation implies  $\bar{Q}Q$  condensation as the origin of EWSB, and a strongly coupled “Higgs” sector would emerge, with a corresponding spectrum of heavy  $\bar{Q}Q$  states. This view may become relevant once the current hint of a 125 GeV light Higgs boson is disproved by experiment, which can happen by the end of 2012.



**Note Added.** Since submission of our work, the CMS experiment has published the bound of  $m_{b'} > 611$  GeV [30], which is now above the nominal unitarity bound. Further studies of the “bootstrap” gap equation reveals the importance of including wave function renormalization, as well as finding a seeming error in the work of Hung and Xiong, leading to considerably larger critical quark mass for EWSB [31].

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