Analysis of the Shape Isomer Yields of ^{237}Np in the Framework of a Dynamical-Statistical Model

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In the framework of the Combined Dynamical Statistical Model, we have analyzed experimental data on the shape isomer yield in $p + {}^{238}$U reactions at $E_p^{\mathrm{lab}} = 9.75 - 12.5$ MeV and obtained information on the double-humped fission barrier parameters for some Np isotopes. Our analysis shows that the depth of the second potential well is less than those from the statistical model calculations.

I. INTRODUCTION

At present the double-humped fission barrier concept has firmly established itself in nuclear physics. The second minimum in the potential energy surface of heavy nuclei was established by Strutinsky's shell-correction method. Therefore, for heavy nuclei two classes of excited states exist, which differ in the value of deformation when the fission barrier is double-humped. The existence of a second well in the fission barrier explains the nature of spontaneously fissionable isomers, the resonance effects in sub-barrier fission, and other phenomena. Furthermore the existence of two classes of excited states for heavy nuclei considerably affects the decay time of such systems. In the present work we use a dynamical-statistical model of induced fission [1] to analyze the experimental fission probabilities of some Np isotopes and shape isomer yields for the ^{238}U(p,2n)^{237}Np reaction at $E_p^{\mathrm{lab}} = 9.75 - 12.5$ MeV.

It should be stressed that so far experimental data on shape isomer yields for ^{237}Np have only been analyzed within the statistical theory of nuclear reactions.

II. DETAILS OF THE MODEL AND ANALYSIS OF THE EXPERIMENTAL DATA

In the framework of the Dynamical-Statistical model we combine a dynamical and statistical description of heavy ion induced fission, so that in the first potential well we use the statistical model and at each time $\hbar/(\Gamma_n + \Gamma_p + \Gamma_\alpha + \Gamma_\gamma)$ calculate the decay widths for the emission of n, p, α, γ and the width of the decay channel related to passing the inner fission barrier. The probabilities of decay via different channels can be calculated by using a standard Monte Carlo cascade procedure where the kind of decay is selected with the weights $\Gamma_i/\Gamma_{\mathrm{tot}}$ ($i = n, p, \alpha, \gamma$, fission) and $\Gamma_{\mathrm{tot}} = \Gamma_n + \Gamma_p + \Gamma_\alpha + \Gamma_\gamma$. This procedure simulates the law for radioactive decay for the different particles. After each emission act of a particle of kind ν the kinetic energy of the emitted particle is calculated by a hit and miss

Monte Carlo procedure. The loss of angular momentum is taken into account by assuming that a neutron carries away 1 $\hbar$, a proton 1 $\hbar$, an α-particle 2 $\hbar$, and a γ quantum 1 $\hbar$. This procedure allows for multiple emissions of light particles and a higher chance of fission. If a random choice of a decay channel leads to the transition of the nucleus from the first potential well to the second one, further evolution of the nucleus is simulated in terms of the coupled Langevin equations. It should be stressed that simulation of the fission process of a nucleus in terms of Langevin equations also allows for the emission of n, p, α, and γ quanta. The result of the simulation of the nucleus evolution in the second potential well can be classified as follows: 1) overcoming the second barrier and reaching the scission point; 2) population of the second potential well and cooling there via particle or γ emission, so that this event is interpreted as the formation of shape isomers; 3) returning of the system into the first potential well.

In order to specify the shape collective coordinates for a dynamical description of nuclear fission, we use the shape parameters r, h, α as suggested by Brack *et al.* [2]. However, we simplify the calculation by considering only symmetric fission and further neglect the neck degree of freedom. Therefore, the coupled Langevin equations in one dimension can be written [3] as

$$
\begin{aligned}
\frac{dp}{dt} &= \frac{1}{2} \left(\frac{p}{m(r)} \right)^2 \frac{dm(r)}{dr} - \frac{dV}{dr} - \beta(r)p + f(t), \\
\frac{dr}{dt} &= \frac{p}{m(r)}.
\end{aligned}
\tag{1}
$$

Here f is a random force with an amplitude $\eta(Tm\beta)^{1/2}$, and η is a random number with the following properties $\langle \eta(t) \rangle = 0$ and $\langle \eta(t) \eta(t') \rangle = 2\delta(t - t')$.

The fission mode damping parameter β is taken to depend on the expression $\beta \approx 0.6T^2 / \left(1 + T^2/40\right)$, which is also a good approximation of the microscopic calculations carried out in terms of the linear response theory.

The initial values of r and p are simulated by the Neumann method with the generating function

$$
\Phi(r, p, J) \propto \frac{\rho_{1f} \left(E^* - B_f^1 - \varepsilon, J \right)}{1 + \exp \left(-\frac{2\pi\varepsilon}{\hbar\omega_{1f}} \right)} \delta(r - r_{1f}),
\tag{2}
$$

where E^* is the excitation energy of the fissioning nucleus, $\varepsilon = p^2/2m$ is the kinetic energy of the collective motion, $\rho_{1f}(E)$ is the level density at the first saddle point, B_f^1 is the inner fission barrier height, and ω_{1f} and r_{1f} are the fission barrier curvature and the collective coordinate at the first saddle point, respectively.

The collective inertia, m, is calculated in the frame of the Werner–Wheeler approach and T is the nuclear temperature that is defined as

$$
T = (E_{\text{int}} /a(r))^2
\tag{3}
$$

with

$$E_{\text{int}} = E^* - p^2/(2m) - V(r, T, J) - E_{\text{rot}}(J), \tag{4}$$

where $E_{\text{rot}}(J)$ and $a(r)$ are the rotational energy and the level density parameter, respectively.

The coordinate dependent level density parameter is of the form

$$a(q) = a_v A + a_s A^{2/3} B_s(q), \tag{5}$$

where A is the mass number of the compound nucleus, $a_v = 0.073$ MeV^{-1}, $a_s = 0.095$ MeV^{-1}, and B_s is the dimensionless functional of the surface energy in the LDM.

The potential energy of a fissionable nucleus is calculated as the sum of the liquid drop potential energy $V_{\text{ld}}(r, J)$ of a rotating nucleus with an angular momentum J and a shell correction δw:

$$V(r, J, T) = V_{\text{ld}}(r, J) + \delta w(r) \times \left[1 + \exp\left(\frac{T - T_0}{d}\right)\right]^{-1}, \tag{6}$$

where r is the distance between the centers of mass of the forming fission fragments, and the bracketed expression describes the damping of the shell effects with the growth of temperature T. The values of the parameters $T_0 = 1.75$ MeV and $d = 0.2$ MeV are taken from Ref. [4].

To calculate the shell correction as a function of the deformation r, the double-humped fission barrier can be approximated by smoothly joined parabolas [5], and, in the zero temperature limit, $\delta w(r)$ is equal to the difference between the above approximation and $V_{\text{ld}}(r, J = 0)$. The value of the equilibrium deformation and corresponding shell correction are taken from [6], and the position of the inner- fission barrier from [7], other parameters of the double-humped fission barriers (the second potential well depth, inner barrier height and outer barrier height) are taken to be free.

Figure 1 shows the results of calculation of the double-humped fission barrier and the shell correction for ^{239}Np, ^{238}Np, and ^{237}Np.

The particle emission width of a particle of kind ν is given by Ref. [8]:

$$\Gamma_v = (2s_v + 1)\frac{m_v}{\pi^2 \hbar^2 \rho_c(E_{\text{int}})} \times \int_0^{E_{\text{int}} - B_v} d\varepsilon_v \rho_R(E_{\text{int}} - \varepsilon_v)\varepsilon_v \sigma_{\text{inv}}(\varepsilon_v), \tag{7}$$

where s_v is the spin of the emitted particle ν and m_ν is its reduced mass with respect to the residual nucleus. $\rho_c(E_{\text{int}})$ and $\rho_R(E_{\text{int}} - \epsilon_\nu)$ are the level densities of the compound and residual nuclei.

The variable ϵ_ν is the kinetic energy of the evaporated particle ν. The intrinsic energy and liquid drop binding energies are denoted by E_{int} and B_v. The inverse cross sections can be written as [9]

$$\sigma_{\text{inv}}(\varepsilon_v) = \begin{cases} \pi R_\nu^2 \left(1 - V_\nu/\varepsilon_\nu\right) & \text{for } \varepsilon_\nu > V_\nu, \\ 0 & \text{for } \varepsilon_\nu < V_\nu, \end{cases} \tag{8}$$
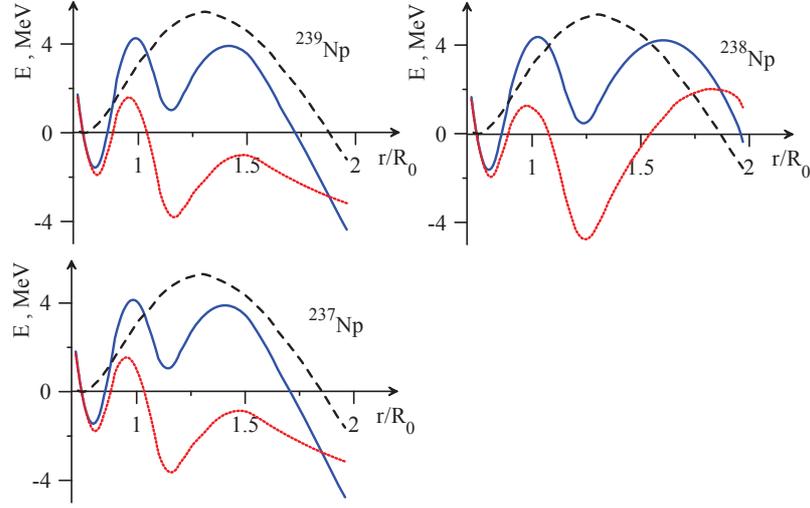
FIG. 1: Calculated double-humped fission barrier (solid curve), liquid-drop fission barrier (dashed curve), and shell correction (dotted curve) for ^{239}Np, ^{238}Np, and ^{237}Np.

with

$$R_\nu = 1.21 \left[(A - A_\nu)^{1/3} + A_\nu^{1/3} \right] + \left(3.4 \big/ \varepsilon_\nu^{1/2} \right) \delta_{\nu,n}, \tag{9}$$

where A_ν is the mass number of the emitted particle $\nu = n, p, \alpha$. The barriers for the charged particles are

$$V_\nu = \left[(Z - Z_\nu) Z_\nu K_\nu \right] / (R_\nu + 1.6), \tag{10}$$

with $K_\nu = 1.32$ for α and the deuteron and 1.15 for the proton.

The width of the gamma emission is calculated by the following formula [10]:

$$\Gamma_\gamma \cong \frac{3}{\rho_c(E_{\text{int}})} \int_0^{E_{\text{int}}} d\varepsilon \rho_c(E_{\text{int}} - \varepsilon) f(\varepsilon), \tag{11}$$

here ϵ is the energy of the emitted γ quanta and $f(\epsilon)$ is defined by

$$f(\varepsilon) = \frac{4}{3\pi} \frac{e^2}{\hbar c} \frac{1+k}{mc^2} \frac{NZ}{A} \frac{\Gamma_G \varepsilon^4}{(\Gamma_G \varepsilon)^2 + (\varepsilon^2 - E_G^2)^2}, \tag{12}$$

with $E_G = 80 A^{-1/3}$, $\Gamma_G = 5$ MeV, and $k = 0.75$ [11], E_G and Γ_G are the position and width of the giant dipole resonance, respectively. The widths of the decay channels related to passing the inner and outer fission barriers are calculated by the Bohr–Wheeler relations using Kramer's correction [12]. Level densities are calculated by considering the paring correlations, collective vibrations, and rotation in the nuclei in the adiabatic approximation as in Refs. [13, 14]. It should be stressed that the level density is a key physical quantity in
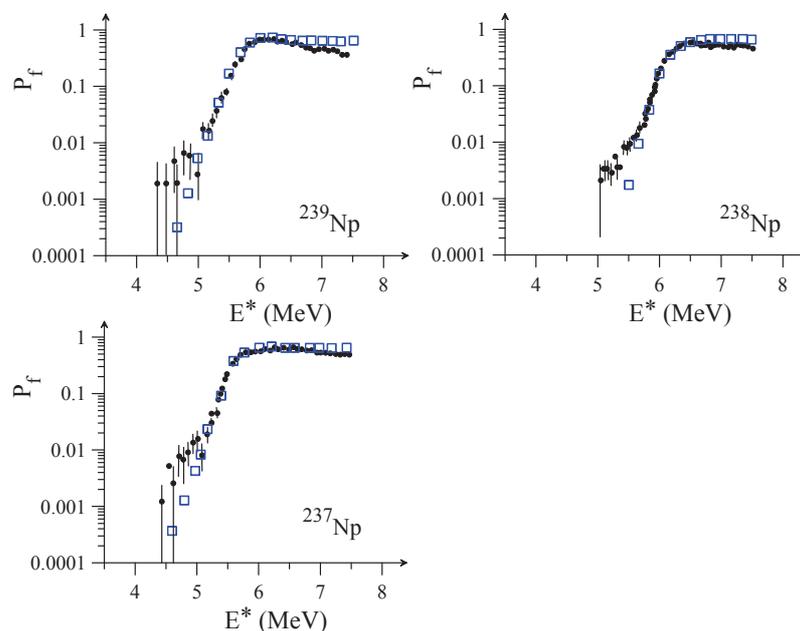
FIG. 2: Fission probability for ^{239}Np, ^{238}Np, and ^{237}Np: □ calculated data, ● experimental data [15].

the calculations because it comes in all the disintegration widths, i.e., evaporation, fission, and gamma emission.

The height and curvature of the inner fission barrier can be determined from the condition of the best fit to the experimental data on fission probabilities.

Figure 2 shows the fission probability distributions for the ^{239}Np, ^{238}Np, and ^{237}Np isotopes.

The depth of the second potential well and the height of the outer fission barrier are found from the condition of the best fit to the experimental values of the ^{237}Np shape isomer yield. Figure 3 shows the results of calculating the shape isomer yield for ^{237}Np.

In Table I the parameters calculated in this investigation for the double-humped fission barriers are compared with well-known reference data [17].
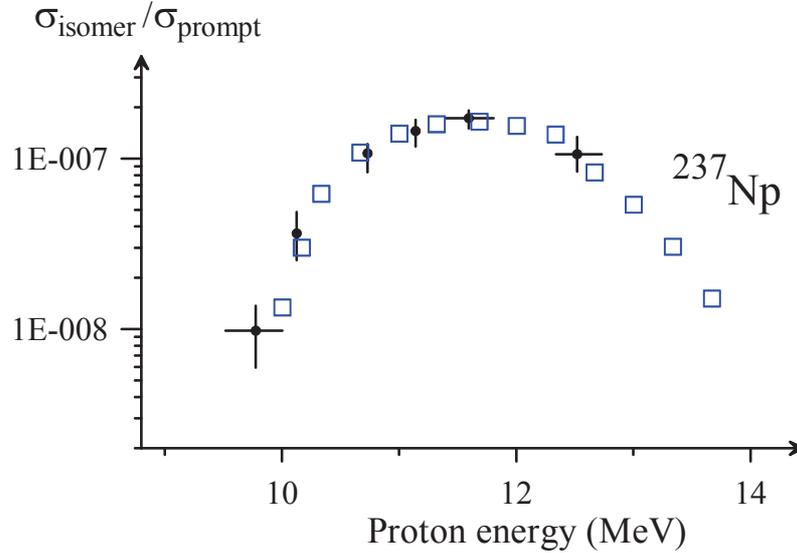
FIG. 3: Yield of the ^{237}Np shape isomer for the $p+^{238}U$ reaction, $\square$ calculating data, $\bullet$ experimental data [16].

TABLE I: Parameters of the double-humped fission barrier for Np isotopes.

Nucleus	Inner barrier height, MeV		Outer barrier height, MeV		Depth of the second well, MeV	
^{237}Np	5.60[a]	5.70 ± 0.20[b]	5.35[a]	5.40 ± 0.20[b]	2.50[a]	2.80 ± 0.30[b]
^{238}Np	6.00[a]	6.10 ± 0.20[b]	5.85[a]	6.00 ± 0.20[b]	2.10[a]	2.30 ± 0.20[b]
^{239}Np	5.85[a]	5.90 ± 0.20[b]	5.50[a]	5.40 ± 0.20[b]	2.60[a]	–

[a] Present study
[b] Reference data [17]

III. SUMMARY AND CONCLUSIONS

A dynamical statistical model was used to analyze the experimental shape isomer yield data in the reaction $p + ^{238}U$ at $E_p^{lab} = 9.75 - 12.5$ MeV. In terms of this analysis we obtained information on the fission barrier parameters of some Np isotopes.

It should be stressed that so far experimental data on shape isomer yields for ^{237}Np have only been analyzed within the statistical theory of nuclear reactions, but in this research we considered the effect of collective motion, and have shown that the depth of the second potential well should be less than the results of the statistical model calculations.

Acknowledgements

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